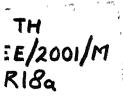
# AN APPROACH TO FREQUENCY BASED GENERATION COST COMPUTATION

by M KRISHNA RAO





DEPARTMENT OF ELECTRICAL ENGINEERING/ACES
INDIAN INSTITUTE OF TECHNOLOGY, KANPUR
February, 2001

# AN APPROACH TO FREQUENCY BASED GENERATION COST COMPUTATION

A Thesis submitted in partial fulfilment of the requirement for the degree of

#### **MASTER OF TECHNOLOGY**

by M KRISHNA RAO 9910444



to the

Department of Electrical Engineering/ACES
INDIAN INSTITUTE OF TECHNOLOGY, KANPUR

February 2001



#### **CERTIFICATE**

This is to certify that the work contained in this thesis titled AN AFPROACH TO EASE GENERATION LOST COMPUTATION IN Submitted by M Krishna Rao has been carried out under my supervision and that has not been submitted elsewhere for a degree.

Feb 2001

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#### **ABSTRACT**

Many countries in the world have opened their power markets to allow the competition between the power producers. The affect of the load changes on the operating frequency and real power output of the generator have been studied in the present work. If the frequency goes out of the limits the generator may loose synchronism. The frequency of the system should be maintained with in acceptable limits. In an interconnected power system, change in load of one control area will also affect the frequency and generation of the generators in other control areas. The variation in generation cost because of these deviations in frequency and real power output of the generator have been studied for a single control area case. The excessive generation cost because of the load change in this control area can be charged to that particular user. The powerful Newton-Raphson method has been used for solving the load flow problem. The real power generation has been optimized using optimal power flow. Load frequency control has been used to calculate the frequency deviations.

# **INDEX**

	List o	f Figure	es	
	List o	f Tables	S	
1.	Introd	luction		1
	1.1	Gener	ral	1
	1.2	Thesi	s Organization	2
2.	Load	Flow ar	nd Optimal Power Flow solutions for various test systems	3
	2.1	Introd	uction	3
		2.1.1	Load Flow Solution	3
		2.1.2	Optimal Power Flow	5
	2.2	Newto	on-Raphson Load Flow (NRLF) Method	6
		2.2.1	Calculation of Jacobian	6
		2.2.2	Iterative Algorithm for NR method	8
		2.2.3	Adjustments in NRLF method	8
			2.2.3.1 Constraints on Reactive Power	9
			2.2.3.2 Constraints on Bus Voltages	9
		2.2.4	Flow chart for NRLF method	10
	2.3	Optim	nal Power Flow	12
		2.3.1	Generator Operating Cost	12
		2.3.2	Optimal Load Flow Solution	12
			2.3.2.1 OPF without inequality constraints	13
			2.3.2.2 Computational Procedure	15
			2.3.2.3 Inequality Constraints on Control Variables	17
	2.4	Result	ts	19
		2.4.1	Results for 43 bus test system	19
		2.4.2	Results for 13 bus test system	24
		2.4.3	Results for 14 bus test system	27
		2.4.4	Results for 11 bus test system	30
3	Gener	ration pr	ricing in single area power System	32
	3 1	Introd	uction	32

	3.2	Load frequency control		34
		3.2.1	Model of governor	34
		3.2.2	Model of turbine	34
		3.2.3	Generator load model	35
		3.2.4	Complete block diagram of LFC for single area	36
		3.2.5	Steady state analysis	37
		3.2.6	Dynamic response	38
	3.3	Gener	ration Pricing	39
	3.4	Result	ts	43
		3.4.1	Generation Pricing for 43 bus test system	43
		3.4.2	Generation Pricing for 13 bus test system	49
		3.4.3	Generation Pricing for 14 bus test system	55
		3.4.4	Generation Pricing for 11 bus test system	61
4	Concl	usions a	and Scope for Further Work	67
	4.1	Concl	usions	67
	4.2	Scope	for Further Work	68
A	. 1: A			(()
Appen		40.1		69
			us system	69
	Data f	or 13 bu	us system	73
	Data f	or 14 bu	us system	76
	Data f	or 11 bu	us system	80

82

References

# LIST OF FIGURES

Fig 2.1	Flow Chart for NRLF Method	10
Fig 3.1	Model of governor	34
Fig 3.2	Model of turbine	35
Fig 3.3	Model of Generator-load	36
Fig 3.4	Block diagram representation of load frequency control	37
Fig 3.5	Load frequency control with PID controller	39
Fig 3.6	Frequency Deviation in 43 power system after a load change	
	of 0.05 p.u at bus 3	44
Fig 3.7	Change in Generation of 43 power system after a load change	
	of 0.05 p.u at bus 3	45
Fig 3.8	Total Generation in 43 bus power system after a load change	
	Of 0.05 p.u at bus 3	46
Fig 3.9	Frequency of 43 bus power system after a load change of	
	0.05 p.u at bus 3	47
Fig 3.10	Generation Cost in 43 bus power system after a load change	
	of 0.05 p.u at bus 3	48
Fig 3.11	Frequency Deviation in 13 power system after a load change	
	of 0.02 p.u at bus 12	50
Fig 3.12	Change in Generation of 13 power system after a load change	
	of 0.02 p.u at bus 12	51
Fig 3.13	Total Generation in 13 bus power system after a load change	
	of 0.02 p.u at bus 12	52
Fig 3.14	Frequency of 13 bus power system after a load change of	
	0.02 p.u at bus 12	53
Fig 3.15	Generation Cost in 13 bus power system after a load change	
	of 0.02 p.u at bus 12	54
Fig 3.16	Frequency Deviation in 14 power system after a load change	
	of 0.05 p.u at bus 9	56

Fig 3.17	Change in Generation of 14 power system after a load change	
	of 0.05 p.u at bus 9	57
Fig 3.18	Total Generation in 14 bus power system after a load change	
	of 0.05 p.u at bus 9	58
Fig 3.19	Frequency of 14 bus power system after a load change of	
	0.05 p.u at bus 9	59
Fig 3.20	Generation Cost in 14 bus power system after a load change	
	of 0.05 p.u at bus 9	60
Fig 3.21	Frequency Deviation in 11 power system after a load change	
	of 0.05 p.u at bus 5	62
Fig 3.22	Change in Generation of 11 power system after a load change	
	of 0.05 p.u at bus 5	63
Fig 3.23	Total Generation in 11 bus power system after a load change	
	Of 0.05 p.u at bus 5	64
Fig 3.24	Frequency of 11 bus power system after a load change of	
	0.05 p.u at bus 5	65
Fig 3.25	Generation Cost in 11 bus power system after a load change	
	of 0.05 p.u at bus 5	66
Fig A.1	43 bus power system	69
Fig A.2	13 bus power system	73
Fig A.3	14 bus power system	76
Fig A.4	11 bus power system	80

# LIST OF TABLES

Table 2.1	Load Flow Solution for 43 bus test system using NRLF	
	Method without any constraints	20
Table 2.2	Load Flow Solution for 43 bus test system using NRLF	
	Method with the constraints on Bus Voltages	21
Table 2.3	Voltage Magnitude and Phase Angles at each bus after OPF	
	for 43 bus test system with constraints	22
Table 2.4	Load Flow Solution for 13 bus test system using NRLF	
	Method without any constraints	24
Table 2.5	Load Flow Solution for 13 bus test system using NRLF	
	Method with the constraints on Bus Voltages	25
Table 2.6	Voltage Magnitude and Phase Angles at each bus after OPF	
	for 13 bus test system with constraints	26
Table 2.7	Load Flow Solution for 14 bus test system using NRLF	
	Method without any constraints	27
Table 2.8	Load Flow Solution for 14 bus test system using NRLF	
	Method with the constraints on Bus Voltages	28
Table 2.9	Voltage Magnitude and Phase Angles at each bus after OPF	
	for 14 bus test system with constraints	29
Table 2.10	Load Flow Solution for 11 bus test system using NRLF	
	Method without any constraints with a tolerance of 0.008	30
Table 2.11	Load Flow Solution for 11 bus test system using NRLF	
	Method with the constraints on Bus Voltages	31
Table 3.1	Comparison of the frequency, real power output and generation	
	Cost before and after disturbance for 43 bus test system	48
Table 3.2	Comparison of the frequency, real power output and generation	
	Cost before and after disturbance for 13 bus test system	54
Table 3.3	Comparison of the frequency, real power output and generation	
	Cost before and after disturbance for 14 bus test system	60

Table 3.4	Comparison of the frequency, real power output and generation	
	Cost before and after disturbance for 11 bus test system	66
Table A.1	Y Bus matrix elements of 43 bus system	70
Table A.2	Operating Condition of 43 bus system	72
Table A.3	Bus data for 13 bus system	74
Table A.4	Line data for 13 bus ill-conditioned system	74
Table A.5	Transformer data for 13 bus system	75
Table A.6	Assumed Values for the cost coefficients	75
Table A.7	Marginal Costs associated with various equipments	75
Table A.8	Generator Model Parameters	75
Table A.9	Generator data for 14 bus system	77
Table A.10	Generator bus voltage for 14 bus system	77
Table A.11	Transformer data for 14 bus system	78
Table A.12	Load bus data for 14 bus system	78
Table A.13	Line data for 14 bus system	79
Table A.14	Bus data for 11 bus system	81
Table A.15	Y bus matrix for 11 bus system	81

# Chapter 1

## INTRODUCTION

# 1.1 General

Small scale power generating technologies are gradually replacing conventional generating technologies in various applications, in the electric power system. These distributed technologies have many benefits, such as high fuel efficiency, short construction lead time, modular installation and low capital expense, which all contribute to their growing popularity [1]. In the last few years many countries in the world have opened their power markets, to allow competition between power producers and at the distribution side also. The industry restructuring process is moving the power sector in general away from the traditional vertical integration and cost-based regulation toward increased exposure to market forces.

The considerations involved in the smooth integration of distributed generation into the distribution system range from long term siting questions to concerns over maintaining frequency stability and the desired voltage profile. Once location and mode of operation are decided, and the necessary protection equipment is installed, the small generators will be able to supply power to customers, whether by contracting directly with customers, a power marketer or the system operator. The modeling for these bulk interactions involves using well established static models such as load flow and optimal flow models.

The load flow solution gives the nodal voltages and phase angles and hence the power injection at all the buses and power flows through interconnecting transmission lines. Load flow solution also gives the initial conditions of the system when the transient behavior of the system is to be studied [2]. The Newton-Raphson method is used for solving the load flow problem.

The optimal power flow uses the load flow solution and gives the solution, which results in optimal cost of generation [3].

Research is going on how the users who cause the frequency changes at the generator can be penalized. Much work has been done in the previous years on the reactive power pricing. This thesis work is started to attain some price model to price the frequency changes at the generator. The load in the power system goes on changing time to time. Following a load change the frequency deviations and the generation changes at every generator are to be calculated. Some kind of control should be there to attain normal frequencies. In the competitive market there can be number of independent producers who will take decisions so as to maximize the profits and to maintain the operating voltage and frequency at the normal value.

Various power systems [4] have been taken for the study. Some of these are ill-conditioned systems as given in [4].

# 1.2 Thesis Organization

The Newton-Raphson Load Flow method has been explained to calculate the voltage and phase angle at all buses for the given load schedule without constraints. Then by taking constraints also the load flow is run. The optimal power flow to optimize the generation obtained through load flow solution has been explained in chapter 2. Various test systems have been taken for the study and load flow solutions for these systems with and without constraints have been shown in chapter 2.

When the load at one bus is increased the frequency and real power generation will also change at the generator. All the generators in every system have been taken as a single control area. Using the load flow method the load at the generator for the new condition has been calculated from which the change in the frequency and real power output of the generator can be calculated. The price model that calculates the change in the generation cost because of the change in the frequency and generation is explained in chapter 3. The generation cost for every test system has been given in chapter 3 considering every system as an isolated control area.

# Chapter 2

# LOAD FLOW AND OPTIMAL POWER FLOW SOLUTIONS FOR VARIOUS TEST SYSTEMS

#### 2.1 INTRODUCTION

### 2.1.1 Load Flow Solution

Load flow solution is a solution of the network under steady state condition subjected to certain inequality constraints under which the system operates [2]. These constraints can be in the form of load bus voltages, reactive power generation of the generators, the tap settings of a tap changing under load transformer etc.

The load flow solution gives the bus voltages and phase angles and hence the power injection at all the buses and power flows through interconnecting transmission lines. Load flow solution is essential for designing a new power system and for planning extension of the existing one for increased load demand. These analyses require the calculation of numerous load flows under both normal and abnormal (outage of transmission line, or outage of some generating source) operating conditions. Load flow solution also gives the initial conditions of the system when the transient behavior of the system is to be studied. Single phase representation is enough under balanced operating conditions. A load flow solution of the power system requires mainly the following steps:

- Formulation of the network equations.
- Suitable mathematical technique for solution of the equations.

Under steady state condition the network equations will be in the form of simple algebraic equations. The load and hence generation are continually changing in a real

power system, but for solving load flow it is assumed that loads and hence generation are fixed at a particular value over a suitable period of time. E.g. half an hour or so.

In a power system each bus or node is associated with four quantities, real and reactive powers, bus voltages magnitude and its phase angle. In a load flow solution two out of four quantities are specified and the remaining two are required to be obtained through the solution of the equations. The buses are classified depending upon the quantities specified into the following three categories

**Load bus**: At this bus the real and reactive components of power are specified. It is desired to find out the voltage magnitude V and phase angle  $\delta$  through the load flow solution. Voltage at load bus can be allowed to vary within the permissible value e.g. 5%.

Generator bus or voltage controlled bus: Here the voltage magnitude corresponding to the generation voltage and real power  $P_G$  corresponding to its ratings are specified. It is required to find out the reactive power generation  $Q_G$  and the phase angle  $\delta$  of the bus voltage.

**Slack, swing or reference bus:** Here the voltage magnitude V and phase angle  $\delta$  are specified. This will take care of the additional power generation required and Transmission losses. It is required to find of real and reactive power generations ( $P_G$ ,  $Q_G$ ) at this bus.

Load flow solution can be achieved with iterative methods. There are many kinds of iterative methods out of which Newton-Raphson method is superior. The second order method is also used in the work for solving the load flow problem.

# 2.1.2 Optimal Power Flow (OPF)

In the load flow problem as explained above, two variables are specified at each bus and the solution is then obtained for the remaining variables. The specified variables are real and reactive powers at PQ buses, real powers and voltages at PV buses, and voltage and angle at the slack bus. The additional variables to be specified for load flow solution are the tap settings of regulating transformers. If the specified variables are allowed to vary in a region constrained by practical considerations (upper and lower limits of real and reactive generations, bus voltage limits and range of transformer settings), these result in infinite number of load flow solutions each pertaining to one set of values of specified variables. The best choice in some sense of the values of specified variables leads to the best load flow solution. Operating economy is naturally predominant in determining the best choice.

The main economic factor in power system operation is the cost of generating real power.

# 2.2 NEWTON-RAPHSON LOAD FLOW (NRLF) METHOD

# 2.2.1 Calculation of Jacobian

For a N-bus power system there will be n equations for real power injection  $P_i$  and n-equations for reactive power injection  $Q_i$ .

$$P_{i} = P_{Gi} - P_{Di} = \sum_{j=1}^{n} [V_{i} V_{j} Y_{y} \cos(\delta_{i} - \delta_{j} - \theta_{y})]$$

$$Q_{i} = Q_{Gi} - Q_{Di} = \sum_{j=1}^{n} [V_{i} V_{j} Y_{y} \sin(\delta_{i} - \delta_{j} - \theta_{y})] \qquad i = 1, 2, 3, ...., N$$

.... (2.1)

The number of equations to be solved depends upon the specifications we have. If the total number of buses is n and number of generator buses is m then the number equations to be solved will be number of known  $P_1$ 's and number of known  $Q_1$ 's. In the above conditions number of known  $P_1$ 's are n-1 and the number of known  $Q_1$ 's are (n-m), therefore the total number of simultaneous equations will be 2\*n-m-1, and number of unknown quantities are also 2\*n-m-1. Unknowns to be find out are power angles ( $\delta$ ) at all the buses except slack (i.e. n-1) and bus voltages (V) at load bus (i.e. n-m). The following method known as Newton-Raphson method is used for solving the unknown quantities. The problem formulation is as follows:

$$\begin{pmatrix} \Delta P \\ \Delta Q \end{pmatrix} = \begin{pmatrix} \frac{\partial P}{\partial \delta} & \frac{\partial P}{\partial V} \\ \frac{\partial Q}{\partial \delta} & \frac{\partial Q}{\partial V} \end{pmatrix} \begin{pmatrix} \Delta \delta \\ \Delta V \end{pmatrix} \qquad \dots (2.2)$$

$$\Delta P_{i} = P_{i}(specified) - P_{i}$$

$$\Delta Q_{i} = Q_{i}(specified) - Q_{i}$$
...(2.3)

Real power terms will be calculated for all the buses except slack bus and reactive power terms will be calculated for all load buses. In the above equation

$$egin{pmatrix} \Delta P \\ \Delta Q \end{pmatrix}$$
 is the mismatch vector  $egin{pmatrix} \Delta \delta \\ \Delta V \end{pmatrix}$  is the correction vector

and

$$J = \begin{pmatrix} \frac{\partial P}{\partial \delta} & \frac{\partial P}{\partial V} \\ \frac{\partial Q}{\partial \delta} & \frac{\partial Q}{\partial V} \end{pmatrix} \text{ is the Jacobian matrix} \qquad \dots (2.4)$$

The elements of the Jacobian matrix can be calculated using the following equations

$$\frac{\partial P_{i}}{\partial \delta_{i}} = -Q_{i} - V_{i}^{2} B_{i}$$

$$\frac{\partial P_{i}}{\partial \delta_{j}} = V_{i} V_{j} Y_{ij} \sin(\delta_{i} - \delta_{j} - \theta_{ij})$$

$$\frac{\partial P_{i}}{\partial V_{i}} = \sum_{j=1}^{n} V_{j} Y_{ij} \cos(\delta_{i} - \delta_{j} - \theta_{ij}) + V_{i} G_{ii}$$

$$\frac{\partial P_{i}}{\partial V_{j}} = V_{i} Y_{ij} \cos(\delta_{i} - \delta_{j} - \theta_{ij})$$

$$\frac{\partial Q_{i}}{\partial \delta_{i}} = P_{i} - V_{i}^{2} G_{ii}$$

$$\frac{\partial Q_{i}}{\partial \delta_{j}} = -V_{i} V_{j} Y_{ij} \cos(\delta_{i} - \delta_{j} - \theta_{ij})$$

$$\frac{\partial Q_{i}}{\partial V_{i}} = \sum_{j=1}^{n} V_{j} Y_{ij} \sin(\delta_{i} - \delta_{j} - \theta_{ij}) + V_{i} B_{ii}$$

$$\frac{\partial Q_{i}}{\partial V_{j}} = V_{i} Y_{ij} \sin(\delta_{i} - \delta_{j} - \theta_{ij})$$
.....(2.5)

Procedure for this iterative method is for the given system first the Y-bus matrix has to be formed.

$$Y = G + j B$$

Where

Y is admittance matrix

G is real part of Y-bus matrix

B is imaginary part of Y-bus matrix

The resistance and reactance of each line have been given for the given system with which the admittance matrix can be calculated.

# 2.2.2 Iterative Algorithm for N-R Method

- 1. With voltage and angle (usually  $\delta = 0$ ) at slack bus fixed, assume voltage magnitude and power angles at PQ buses and  $\delta$  at all PV buses. Generally flat voltage start will be used.
- 2. Compute  $\Delta P_i$  for all buses except slack bus and  $\Delta Q_i$  for all PQ buses using Eq. (2.3). If all the values are less than the prescribed tolerance, stop the iterations.
- 3. If the convergence criterion is not satisfied, evaluate elements of the jacobian using Eq. (2.5)
  - 4. Solve the Eq. (2.2) for correction vector.
- 5. Update voltage angles and magnitudes by adding the corresponding changes to the previous values and return to step 2.

# 2.2.3 Ad'ustments in NRLF Solution

In the present thesis work some constraints have been applied on voltages at the PQ buses, reactive power at the PV buses so as to keep the system healthy.

#### 2.2.3.1 Constraints on reactive power:

Since Q<sub>1</sub> at generator buses is not given, it is calculated in each iteration at all generator buses and it is been checked for the condition

$$Q_{t,\min} < Q_t < Q_{t,\max} \qquad \dots (2.6)$$

If the above mentioned condition satisfied then the i<sup>th</sup> bus will remain as generator bus and there will be no change in the procedure.

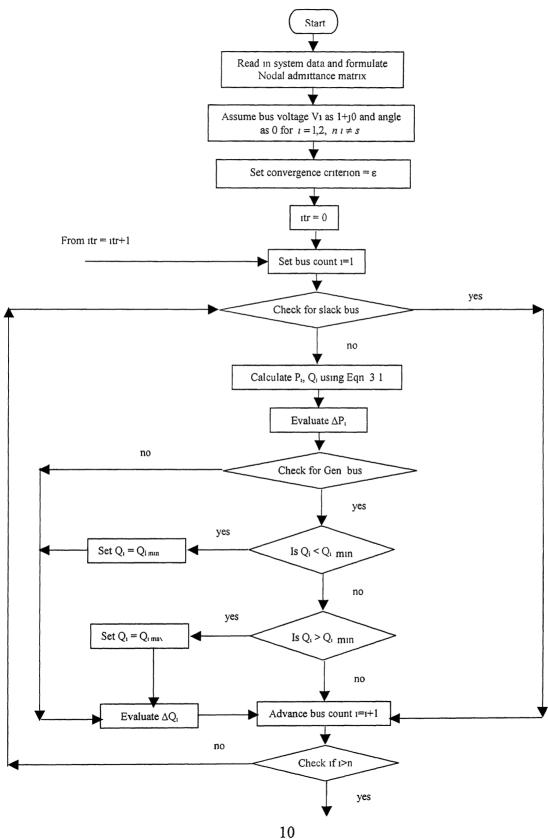
If the condition violates then  $Q_1$  will be set at the limit values it can be either lower limit or upper limit depending upon the violation and i<sup>th</sup> bus will be treated as load bus from the next iteration. Because of this change of bus type one additional equation corresponding to  $\Delta Q_1$  will be added to NRLF scheme.

#### 2.2.3.2 Constraints on bus voltages:

After every iteration voltages and angles are updated with the obtained correction matrix. Voltage has to satisfy the conditioned given below

If it satisfies the condition then there will be no change in the process, if it violates then voltage has to be fixed at one of its lower or upper limits depending upon the violation and the bus type is been changed to generator type. Because of this one equation corresponding to  $\Delta Q_1$  will be deleted from NRLF scheme, so the size of Jacobian matrix will be decreased.

# 2.2.4 Flow Chart for NRLF method



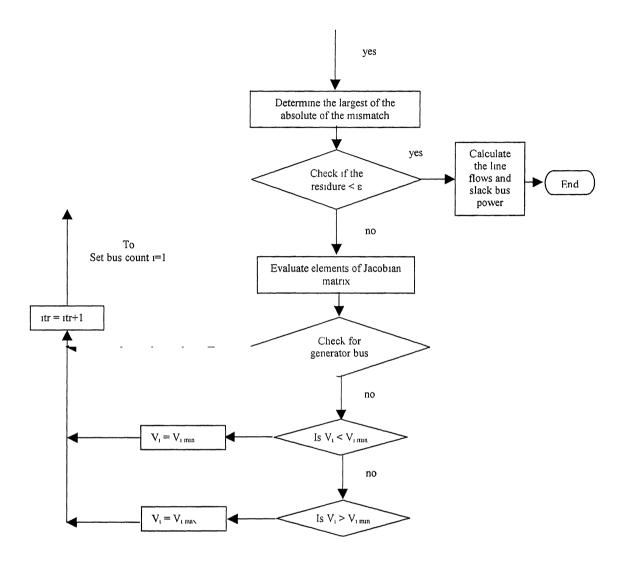


Fig 2.1 Flow chart for load flow solution using Newton-Raphson method

#### 2.3 OPTIMAL POWER FLOW

# 2.3.1 Generator Operating Cost

The major component of generator operating cost is the fuel input/hour, while maintenance contributes only to a small extent. The input-output curve of a unit can be expressed in million kilocalories per hour or directly in terms of rupees per hour versus output in megawatts. For every generator,  $(MW)_{min}$  is the minimum loading limit below which it is uneconomical to operate the unit and  $(MW)_{max}$  is the maximum output limit. The analytical operating cost can be written as  $C_i(P_{G_i})$  Rs/hour at output  $P_{G_i}$ , where the suffix i stands for the unit number. It generally suffices to fit a second degree polynomial [3], i.e.,

$$C_i = \frac{1}{2} a_i P_{Gi}^2 + b_i P_{Gi} + d_i \text{ Rs/hour}$$
 .....(2.8)

The slope of the cost curve, i.e.,  $\frac{dC_i}{dl'_{Gi}}$  is called the incremental fuel cost (IC), and is expressed in units of rupees per megawatt hour (Rs/MWh). If the cost curve is approximated as in Eq. (2.8) then the expression for IC will be

$$(IC)_{i} = a_{i} P_{Ci} + b_{i} \qquad \dots (2.9)$$

i.e., a linear relationship.

# 2.3.2 Optimal Load Flow Solution

The solution technique is based on load flow solution by NR method, a first order gradient adjustment algorithm for minimizing the objective function and use of penalty functions to account for inequality constraints on dependent variables.

### 2.3.2.1 **OPF Without Inequality Constraints**

The objective function to be minimized is the operating cost

$$C = \sum_{i} C_{i} \left( P_{G_{i}} \right) \tag{2.10}$$

subject to the load flow equations

$$P_{i} - \sum_{j=1}^{n} |V_{i}| |V_{j}| |Y_{y}| \cos (\theta_{y} + \delta_{i} - \delta_{j}) = 0$$

$$Q_{i} + \sum_{j=1}^{n} |V_{i}| |V_{j}| |Y_{y}| \sin (\theta_{y} + \delta_{i} - \delta_{j}) = 0$$
for each PQ bus

and

$$P_{i} - \sum_{j=1}^{n} |V_{i}| |V_{j}| |Y_{y}| \cos(\theta_{y} + \delta_{i} - \delta_{j}) = 0 \text{ for each PV bus.} \qquad \dots (2.11)$$

It is to be noted that at the  $i^{th}$  bus

$$P_{i} = P_{Gi} - P_{Di}$$

$$Q_{i} = Q_{Gi} - Q_{Di}$$
.....(2.12)

Where  $P_{D_l}$  and  $Q_{D_l}$  are load demands at bus i.

Equations (2.11) can be expressed in vector form

$$\mathbf{f}(\mathbf{x}, \mathbf{y}) = \begin{bmatrix} P_{i} - \sum_{j=1}^{n} |V_{i}| |V_{j}| |Y_{ij}| \cos(\theta_{ij} + \delta_{i} - \delta_{j}) = 0 \\ Q_{i} + \sum_{j=1}^{n} |V_{i}| |V_{j}| |Y_{ij}| \sin(\theta_{ij} + \delta_{i} - \delta_{j}) = 0 \end{bmatrix}$$
For each PQ bus
$$P_{i} - \sum_{j=1}^{n} |V_{i}| |V_{j}| |Y_{ij}| \cos(\theta_{ij} + \delta_{i} - \delta_{j}) = 0$$
For each PV bus
$$P_{i} - \sum_{j=1}^{n} |V_{i}| |V_{j}| |Y_{ij}| \cos(\theta_{ij} + \delta_{i} - \delta_{j}) = 0$$

.....(2.13)

Where the vector of dependent variables is

$$\mathbf{x} = \begin{bmatrix} |V_i| \\ \delta_i \\ \delta_j \end{bmatrix}$$
 For each PQ bus For each PV bus ...... (2.14)

and the vector of independent variables is

$$\mathbf{y} = \begin{bmatrix} |V_1| \\ S_1 \\ P_i \\ Q_i \\ P_i \end{bmatrix}$$
 For each PQ bus For each PV bus  $|V_i|$ 

$$\mathbf{y} = \begin{bmatrix} \mathbf{u} \\ \mathbf{p} \end{bmatrix} \qquad \dots (2.15)$$

In the above formulation, the objective function should include the slack bus power.

The vector of independent variables  $\mathbf{y}$  can be partitioned into two parts – a vector  $\mathbf{u}$  of control variables, which are to be varied to achieve optimum value of the objective function and a vector  $\mathbf{p}$  of fixed or disturbance or uncontrollable parameters. Control parameters may be voltage magnitudes on PV buses,  $P_{C_i}$  at buses with controllable power etc.

The optimization problem can now be restarted as

subject to equality constraints

To solve the optimization problem, defining the Lagrangian function as

$$\pounds(\mathbf{x}, \mathbf{u}, \mathbf{p}) = C(\mathbf{x}, \mathbf{u}) + \lambda^{T} \mathbf{f}(\mathbf{x}, \mathbf{u}, \mathbf{p}) \qquad \dots (2.18)$$

where  $\lambda$  is the vector of Lagrangian multipliers of the same dimension as f(x, u, p).

The necessary conditions to minimize the unconstrained lagrangian function are

$$\frac{\partial \mathcal{L}}{\partial \mathbf{x}} = \frac{\partial \mathbf{C}}{\partial \mathbf{x}} + \left[ \frac{\partial \mathbf{f}}{\partial \mathbf{x}} \right]^T \lambda = 0 \qquad \dots (2.19)$$

$$\frac{\partial \mathfrak{L}}{\partial \mathbf{u}} = \frac{\partial \mathbf{C}}{\partial \mathbf{u}} + \left[ \frac{\partial \mathbf{f}}{\partial \mathbf{u}} \right]^T \lambda = 0 \qquad \dots (2.20)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = \mathbf{f}(\mathbf{x}, \mathbf{u}, \mathbf{p}) = 0 \qquad \dots (2.21)$$

Where f, x, u, p and  $\lambda$  are vectors.

The Eq. (2.21) is obviously the same as equality constraints. The expressions for

$$\frac{\partial C}{\partial \mathbf{x}}$$
 and  $\frac{\partial \mathbf{f}}{\partial \mathbf{u}}$  as needed in equations (2.19) are rather involved.  $\frac{\partial \mathbf{f}}{\partial \mathbf{x}}$  is nothing but

Jacobian matrix.

Equations (2.19), (2.10) and (2.21) are non-linear algebraic equations and can only be solved iteratively. A simple yet efficient iteration scheme, that can be employed, is the steepest descent method (also called gradient method). The basic technique is to adjust the control vector u, so as to move from one feasible solution point in the direction of steepest descent (negative gradient) to a new feasible solution point with a lower value of objective function. By repeating these moves in the direction of the negative gradient, the minimum will finally be reached.

### 2.3.2.2 Comutational Procedure

The computational procedure is as follows.

- 1. Make an initial guess for **u**, the control variables.
- 2. Find a feasible load flow solution by NR iterative method. The method successfully improves the solution  $\mathbf{x}$  as follows

$$\mathbf{x}^{(r+1)} = \mathbf{x}^{(r)} + \Delta \mathbf{x}$$

where  $\Delta \mathbf{x}$  is obtained by solving the set of linear equations

$$\left[\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\left(\mathbf{x}^{(r)},\mathbf{y}\right)\right]\Delta\mathbf{x} = -\mathbf{f}\left(\mathbf{x}^{(r)},\mathbf{y}\right)$$

or

$$\Delta \mathbf{x} = -(J^{(r)})^{-1} \mathbf{f}(\mathbf{x}^{(r)}, \mathbf{y})$$

the end solution of step 2 are a feasible solution of  $\mathbf{x}$  and the Jacobian matrix.

3. Solve the Eq. (2.19) for

$$\lambda = \left[ \left( \frac{\partial \mathbf{f}}{\partial \mathbf{x}} \right)^T \right]^{-1} \frac{\partial C}{\partial \mathbf{x}} \qquad \dots (2.22)$$

4. Insert  $\lambda$  from Eq. (2.22) into Eq. (2.20), and compute the gradient

$$\nabla \pounds = \frac{\partial \mathbf{C}}{\partial \mathbf{u}} + \left[ \frac{\partial \mathbf{f}}{\partial \mathbf{u}} \right]^{T} \lambda \qquad (2.23)$$

It may be noted that for computing the gradient, the Jacobian  $J = \frac{\partial \mathbf{f}}{\partial \mathbf{x}}$  is already known from the load flow solution.

- 5. If  $\nabla \pounds$  equals zero within prescribed tolerance, the minimum has been reached. Otherwise,
  - 6. Find a new set of control variables

$$\mathbf{u}_{new} = \mathbf{u}_{old} + \Delta \mathbf{u} \qquad \dots (2.24)$$

where

Here  $\Delta \mathbf{u}$  is a step in the negative direction of the gradient. The step size is adjusted by the positive scalar  $\alpha$ .

Selection of  $\alpha$  is the critical part of the algorithm. Too small a value of  $\alpha$  guarantees the convergence but slows down the rate of convergence; too high a value causes oscillations around the minimum.

#### 2.3.2.3 Inequality Constraints on Control Variables

The control variables are always constrained

$$\mathbf{u}_{\min} \leq \mathbf{u} \leq \mathbf{u}_{\max}$$

e.g.

$$P_{G_{l,\min}} \le P_{G_{l}} \le P_{G_{l,\max}} \qquad \dots (2.26)$$

If the correction  $\Delta u_i$  in Eq. (2.25) causes  $u_i$  to exceed one of the limits,  $u_i$  is set equal to the corresponding limit, i.e.,

$$\mathbf{u}_{1,\text{new}} = \begin{cases} u_{1,\text{max}} & \text{if} \quad u_{1,\text{old}} + \Delta u_{1} > u_{1,\text{max}} \\ u_{1,\text{min}} & \text{if} \quad u_{1,\text{old}} + \Delta u_{1} < u_{1,\text{min}} \\ u_{1,\text{old}} + \Delta u_{1} & \text{otherwise} \end{cases}$$

$$(2.27)$$

After a control variable reaches any of the limits, its component in the gradient should continue to be computed in later iterations, as the variable may come within limits at some later stage.

The necessary conditions for minimization of  $\pounds$  under constraint Eq. (2.26) are:

$$\frac{\partial \mathfrak{L}}{\partial u_{i}} = 0 \quad \text{if} \quad u_{i,\min} < u_{i} < u_{i,\max}$$

$$\frac{\partial \mathfrak{L}}{\partial u_{i}} \leq 0 \quad \text{if} \quad u_{i} = u_{i,\max}$$

$$\frac{\partial \mathfrak{L}}{\partial u_{i}} \geq 0 \quad \text{if} \quad u_{i} = u_{i,\min}$$

$$\dots (2.28)$$

Therefore, in step 5 of the computational algorithm, the gradient vector has to satisfy the optimality condition Eq. (2.28).

Once the generation has been decided for the given load schedule the load flow program will be run again after some period as the demand goes on varying. In a power system when the load disturbance occurs the operating frequency and real power output of the generator will change. To determine the change in the load at each generator for the new load schedule again the load flow will be run. These load changes will be the input vector to the load frequency control problem to determine the deviations in the frequency and real power output.

#### 2.4 RESULTS

43 bus [4], 13 bus [4], IEEE 14 bus and 11 bus [4] test systems have been taken for the study. The data is given in the appendix A. The Newton Raphson Load Flow method has been used to solve the load flow problem. The load flow problems have been solved without constraints and with constraints. The optimal power flow has been run to optimize the generation cost. The results of the NRLF method and OPF are shown in the following sections.

# 2.4.1 Results for 43 bus test system

The load flow problem for 43 bus test system [4] has been solved without putting any constraints. The general base case solution is shown in the Table 2.1.

Table 2.1 Load flow solution for 43 bus test system [4] using NRLF method without any constraints

Voltage	Phase Angle
(p.u)	(deg.)
1.136000	0.000000
1.038553	-11.196495
1.015844	-15.235573
0.984997	-13.712234
1.035090	-11 313513
1.037332	-11.568977
1.040850	-15.315901
1.009215	-13.401521
1.007399	-13.422747
1 008570	-13 487796
1.008570	-13.487796
1.043339	-15.553347
0 986969	
	-13.673939
1.009973	-14 641201
1.034337	-11.451182
0.972859	-17.680355
1.008570	-13.487796
1.006835	-15.975134
1.032887	-16.428598
1.045197	-14.405982
0.998752	-13.240570
1.029823	-16.484206
1.006628	-13.304210
1.022492	-17.118298
0.995748	-13.431604
1.010820	-21.370186
1.020050	-15.432568
1.031474	-11.617084
1.003496	-13.178171
0.919032	-11.951977
1.000000	0.692896
1.000000	-6.065698
1.000000	-16.209620
0.845448	-21.630065
1.000000	-15.702249
0.920797	-21.756725
0.975098	-4.542215
0.908746	-21.630065
1.026546	-11.738752
0.913340	-12.175063
1.005302	-14.698162
0.884260	-19.544946
1.024592	-11.763712
d	

The load flow problem has been solved using the first order NRLF method by considering constraints on bus voltages. The minimum and maximum limits have been taken as 0.9 p.u and 1.1 p.u. The solution is given in the Table 2.2.

Table 2.2 Load flow solution using NRLF method for 43 bus test system [4] with the constraints on bus voltages

Voltage	Phase Angle
(p.u)	(deg.)
1.136000	0.000000
1.100000	-10.553851
1.100000	-14.290340
1.052288	-12.973546
1.100000	-10.711639
1.100000	-10.904547
1.100000	-14.274729
1.072088	-12.558681
1.070422	-12.579715
1.071666	-12.644764
1.071666	-12.644764
1.100000	-14.467805
1.053497	-12.906119
1.083747	-13.703580
1.100000	-10.845451
1.041819	-16.320639
1.071666	-12.644764
1.079117	-14.917460
1.100000	-15.236020
1.100000	-13.485208
1.061412	-12.368501
1.100000	-15.374980
1.069123	-12.444268
1.100000	-15.759599
1.060151	-12.609425
1.100000	-19.577074
1.100000	-14.352148
1.100000	-11.058515
1.065570	-12.300383
0.926179	-11.152750
1.000000	0.747890
1.000000	-5.312052
1.000000	-15.323243

0.847790	-20.728674
1.000000	-14.817287
0.923281	-20.854644
0.998606	-4.363649
0.911263	-20.728674
1.095469	-11.168957
0.920555	-11.372754
1.075776	-13.760233
0.893243	-18.610630
1.093655	-11.191561

After getting the load flow solution with constraints on bus voltages the optimal power flow is run for the system to optimize the generation cost. The real power output of each generator is limited to a maximum value of  $(1.1*P_{G_I})$  p.u and to a minimum value of  $(0.9*P_{G_I})$  p.u. The solution giving the bus voltages and phase angles at each bus after optimal power flow considering the limits on the real power output of the generator is shown in Table 2.3.

Table 2.3 Voltage magnitudes and Phase Angles at each bus after Optimal Power Flow for 43 bus test system [4] with constraints

Voltage (p.u)	Phase Angle (deg.)
1.136000	0.000000
1.100000	-9.787865
1.100000	-12.938932
1.048144	-11.749151
1.080971	-9.665371
1.099894	-10.102128
1.100000	-12.919196
1.064307	-11.284862
1.062623	-11.305931
1.063859	-11.370980
1.063859	-11.370980
1.100000	-13.308905
1.049021	-11.676514
1.083086	-12.910483
1.099966	-10.064411

1.033322	-15.105379
1.063859	-11.370980
1.074265	-13.705566
1.100000	-14.013776
1.100000	-12.440043
1.055842	-11.114266
1.099511	-14.126218
1.062189	-11.181230
1.100000	-14.182029
1.055060	-11.368999
1.100000	-17.908845
1.100000	-12.944973
1.099240	-10.262341
1.059518	-11.048646
0.925502	-9.940663
1.000022	2.063675
1.000036	-4.097875
1.000001	-14.172171
0.847557	-19.559920
1.000003	-13.657221
0.923035	-19.685959
0.996339	-3.054983
0.911013	-19.559920
1.094849	-10.372876
0.919872	-10.160957
1.075143	-12.967147
0.892393	-17.411172
1.093034	-10.395500

# 2.4.2 Results for 13 bus test system

The load flow problem for 13 bus test system [4] has been solved without any constraints. The general base case solution is shown in the Table 2.4.

Table 2.4 Load flow solution for 13 bus test system [4] using NRLF method without any constraints

Voltage (p.u)	Phase Angle (deg.)
1.000000	0.000000
1.063717	1.347255
1.179639	1.945324
1.160323	1.984012
1.000000	2.110602
1.037000	9.211764
1.063684	8.365741
1.100000	7.426077
0.943000	13.840178
1.100000	7.826597
0.957134	11.835802
1.036184	7.363695
0.969457	4.326548

The load flow problem has been solved using the first order NRLF method by puttiing constraints on bus voltages. The minimum and maximum limits have been taken as 0.9 p.u and 1.1 p.u. The solution is given in the Table 2.5.

Table 2.5 Load flow solution using NRLF method for 13 bus test system [4] with the constraints on bus voltages

Voltage (p.u)	Phase Angle (deg.)
1.000000	0.000000
1.063578	1.387019
1.100000	2.247327
1.100000	2.247327
1.000000	2.248149
1.037000	9.473620
1.063746	8.679546
1.100000	7.807725
0.943000	14.221826
1.100000	8.208245
0.957134	12.217450
1.036184	7.745343
0.969457	4.708196

After getting the load flow solution with constraints on bus voltages the optimal power flow is run for the system to optimize the generation cost. The real power output of each generator is limited to a maximum value of  $(1.1*P_{Gl})$  p.u and to a minimum value of  $(0.9*P_{Gl})$  p.u. The solution giving the bus voltages and phase angles at each bus after optimal power flow considering the limits on the real power output of the generator is shown in Table 2.6.

Table 2.6 Voltage magnitudes and Phase Angles at each bus after Optimal Power Flow for 13 bus test system [4] with constraints

Voltage (p.u)	Phase Angle (deg.)
1.000000	0.000000
1.064659	1.239276
1.100000	1.987836
1.099962	1.987937
1.000038	1.988586
1.040593	8.466641
1.062673	7.757496
1.092275	6.959252
0.967112	12.347970
1.100000	7.084377
0.989823	10.584633
1.030016	6.784220
0.928995	4.103847

# 2.4.3 Results for IEEE 14 bus test system

The load flow problem for IEEE 14 bus test system has been solved without any constraints. The general base case solution is shown in the Table 2.7.

Table 2.7 Load flow solution for 14 bus test system using NRLF method without any constraints

Voltage (p.u)	Phase Angle (deg.)
1.060000	0.000000
1 045000	-4.517426
1 010000	-11.962797
1.021593	-9.367419
1.027582	-7.874867
1.070000	-11.655461
1.062376	-12.008487
1.090000	-12.008487
1.055376	-13.383275
1.050294	-13.365262
1.056302	-12.648834
1.055290	-12.580481
1.050081	-12.733563
1.034994	-14.102923

The load flow problem has been solved using the first order NRLF method by puttiing constraints on bus voltages. The minimum and maximum limits have been taken as 0.9 p.u and 1.1 p.u. The solution is given in the Table 2.8.

Table 2.8 Load flow solution using NRLF method for IEEE 14 bus test system with the constraints on bus voltages

Voltage (p.u.)	Phase Angle (deg.)
1.060000	0.000000
1 019702	-4.209865
0.980821	-12.077397
0.988329	-9.248864
0.995728	-7.651070
1.000943	-11.613080
1.018032	-12.217016
1.047298	-12.217016
1.004776	-13.786931
0.995999	-13.738793
0.994520	-12.850663
0.98661 3	-12.671585
0.982425	-12.880401
0.975829	-14.525790

After getting the load flow solution for the problem with constraints on bus voltages the optimal power flow is run for the system to optimize the generation cost. The real power output of each generator is limited to a maximum value of  $(1.1*P_{Gl})$  p.u and to a minimum value of  $(0.9*P_{Gl})$  p.u. The solution giving the bus voltages and phase angles at each bus after optimal power flow considering the limits on the real power output of the generator is shown in Table 2.9.

Table 2.9 Voltage magnitudes and Phase Angles at each bus after Optimal Power Flow for IEEE 14 bus test system with constraints

Voltage (p.u)	Phase Angle
	(deg.)
1.060000	0.000000
1.044759	-4.517426
1.009980	-11.962797
1.021593	-9.367419
1.027582	-7.874867
1.070003	-11.655461
1.062376	-12.008487
1.090001	-12.008487
1.055376	-13.383275
1.050294	-13.365262
1.056302	-12.648834
1.055290	-12.580481
1.050081	-12.733563
1.034994	-14.102923

# 2.4.4 Results for 11 bus test system

The load flow problem for 11 bus test system [4] has been solved without any constraints. The method has diverged for a tolerance level of 0.0001. It has converged when the tolerance is taken as 0.008. But the values obtained are not practical. Practically 11 bus test system diverging when there are no constraints on voltages.

The load flow problem has been solved using the first order NRLF method by considering constraints on bus voltages. The minimum and maximum limits have been taken as 0.9 p.u and 1.1 p.u. The solution is given in the Table 2.11.

Table 2.11 Load flow solution using NRLF method for 11 bus test system [4] with the constraints on bus voltages

Voltage (p.u)	Phase Angle (deg.)
1.024000	0.000000
1.066178	-2.903824
1.056163	-4.662305
1.041775	-3.852663
1.044332	-5.537236
1.061867	-3.934891
0.900000	-24.872891
0.900000	-23.957400
0.900000	-23.878411
0.900000	-36.085608
0.900000	-37.681380

## Chapter 3

# GENERATION PRICING IN SINGLE AREA POWER SYSTEM

## 3.1 INTRODUCTION

Power being a complex quantity, it has been observed that any change in real power output of the generators directly affect the system frequency and tie-line loadings while any change in reactive power affect only the system voltage. This property helps in dividing the control problem of a power system into two separate channels: the MW-frequency control channel and the MVAR-voltage control channel. The control of the real power output of the generators in response to change in system frequency and the tie-line power flow so as to maintain the scheduled system frequency and the tie-line interchange with in permitted limit is termed as Automatic Generation Control (AGC) or Load Frequency Control (LFC) [6]. On the other hand the control of reactive power generation in the system in order to maintain constant voltage is known as excitation Control.

The main objective of the AGC for an inter connected power system can be stated as follows

- 1. Matching generation to load
- 2. Regulating system frequency error to zero
- 3. Distributing generation amongst areas so that inter-area tie-line flows match a prescribed schedule.
- 4. Distributing generation with in each area such that the operating cost of area is minimized.

Automatic Generation Control therefore can be subdivided into two separate control problems. One is, the traditional Load Frequency Control problem, which meets the first three objectives. Second is, Economic Load dispatch problem, which takes care of the last objective.

Generally objective (1) is achieved by the system governor. However, system governor alone is found inadequate to take care of objectives (2) and (3). Therefore, supplementary control is added to the system governor utilizing PID controller such that the deviations in frequency and tie-line loading from the prescheduled values, following a sudden load change in any area, are reduced to zero in the time span of less than a minute. The controller design should be such that not only the objectives (2) and (3) should be met but also the transient oscillations in frequency are kept to a minimum. In a competitive market there will be independent generators in a large number. Every producer will take his own decisions while supplying power to the corresponding consumers. He will fix the cost per unit generation depending upon the conditions in which the generators are being operated and the changes in the demand. In every aspect he will try to increase his profit and at the same time he will keep in mind the competition from the other producers. Industry restructuring, and particularly the deregulation of generation, is opening the power sector to market forces.

To coordinate distributed generator actions in the short-term operations and control of the spot energy and ancillary services markets, a price model is proposed. The method used is developed to accurately capture the cost associated with local deviations from the scheduled power and energy. A mathematical framework for price model, designed to coordinate distributed generators as they participate in both the short run energy market and the ancillary services market. This mathematical framework will take care of both frequency change as well as load change.

# 3.2 LOAD FREC UENCY CONTROL

# 3.2.1 Model of Governor

The speed governing system can be modeled as in Fig 3.1

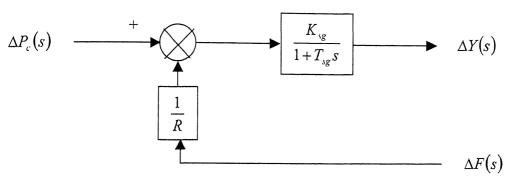


Fig. 3.1 Model of Speed Governor

Where

R=speed regulation of the governor

 $K_{sg}$  = gain of speed governor

 $T_{sy}$  =time constant of speed governor

 $\Delta p_{\ell}$  is the commanded increase in power. This signal sets into motion a sequence of events. When the load increases the frequency goes down. As a result turbine generator speed decreases. Then this signal comes into picture to increase the steam input thus increasing speed, which in turn increases, the frequency.

 $\Delta Y$  is the output signal of the governor. i.e., input signal to the turbine.

## 3.2.2 Model of Turbine

The model requires a relation between changes in power output of the steam turbine to changes in its steam valve opening. A non-reheat turbine with a single gain

factor  $K_i$  and a single time constant  $T_i$  is considered. Thus the transfer function of the turbine is

$$G_{t}(s) = \frac{K_{t}}{1 + T_{t}s} \qquad \dots (3.1)$$

The block diagram representation is shown in Fig 3.2

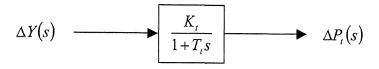


Fig. 3.2 Model of Turbine

#### 3.2.3 Model of Generator and Load

The increment in power input to the generator-load system is

$$\Delta P_G - \Delta P_D$$

where  $\Delta P_{i} = \Delta P_{i}$ , incremental turbine power output and  $\Delta P_{D}$  is the load increment.

As the frequency changes, the motor load changes being sensitive to speed, the rate of change of load with respect to frequency, i.e.,  $\partial P_D/\partial f$  can be regarded as nearly constant for small changes in frequency  $\Delta f$  and can be expressed as

$$(\partial P_D/\partial f)\Delta f = B\Delta f$$

where the constant B can be determined empirically. B is positive for a motor load.

Writing the power balance equation,

$$\Delta P_{c_i} - \Delta P_D = \frac{2HP_r}{f^0} \frac{d}{dt} (\Delta f) + B \Delta f \qquad (3.2)$$

$$\Delta P_{G}(pu) - \Delta P_{D}(pu) = \frac{2H}{f^{0}} \frac{d}{dt} (\Delta f) + B(pu) \Delta f \qquad \dots (3.3)$$

Taking Laplace transform,

$$\Delta F(s) = \frac{\Delta P_{cr}(s) - \Delta P_{D}(s)}{B + \frac{2H}{f^{0}} s} \qquad \dots (3.4)$$

$$\Delta F(s) = \left[\Delta P_{c_i}(s) - \Delta P_{D}(s)\right] \left(\frac{K_{ps}}{1 + T_{ps} s}\right) \qquad \dots (3.5)$$

where

$$T_{ps} = \frac{2H}{Bf^0}$$
 = Power system time constant

$$K_{pv} = \frac{1}{B}$$
 =power system gain

The block diagram representation is shown in Fig 3.3

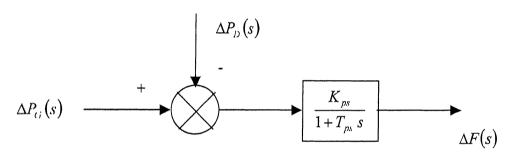


Fig. 3.3 Model of Generator-Load

# 3.2.4 Complete Block Diagram for Single Area Power System

The complete block diagram representation of an isolated power system comprising turbine, generator, governor and load is easily obtained by combining the block diagrams of individual components. The complete block diagram with feedback loop is shown in fig (3.4).

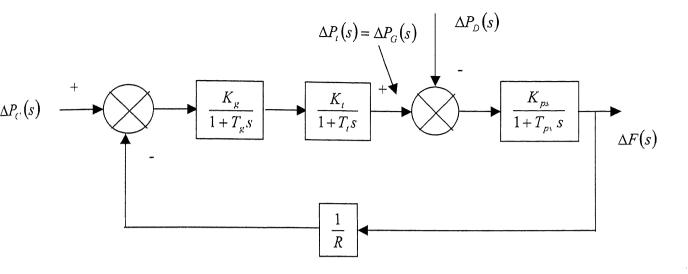


Fig. 3.4 Block diagram representation of Load Frequency Control

#### 3.2.5 Steady State Analysis

Two important incremental inputs to the load frequency control system are -  $\Delta P_C$ , the change in speed changer setting; and  $\Delta P_D$ , the change in load demand. If a simple situation in which the speed changer has a fixed setting (i.e.,  $\Delta P_C = 0$ ) and the load demand changes. This is known as free governor operation. For such an operation the steady state change in system frequency for a sudden load change by an amount  $\Delta P_D$  is

## 3.2.6 Dynamic Res <u>jonse</u>

To obtain the dynamic response giving the change in frequency as function of the time for a step change in load, the Laplace inverse of Eq. (3.6) is to be calculated.

The droop in the frequency from no load to full load should be as small as possible as much change in frequency cannot be tolerated. In fact the steady change in the frequency should be zero. While steady state frequency can be brought back to the scheduled value by adjusting speed changer setting, the system could undergo intolerable dynamic frequency changes with changes in load. It leads to the natural suggestion that the speed changer setting be adjusted automatically by monitoring the frequency changes. For this purpose, a signal from  $\Delta f$  is fed through a PID controller to the speed changer. The above modification makes the steady state error to fall to zero and settling time to reduce.

Therefore,

$$\Delta P_{C}(s) = (-K_1 - K_2/s - K_3 s)b \Delta F(s)$$
 .....(3.8)

where b is feed back gain.

 $K_1$  is gain of proportional controller

 $K_2$  is gain of integral controller

 $K_3$  is gain of differential controller

In this case the change in frequency will be fed back to the PID controller the output of the output of the controller is  $\Delta P_{C}(s)$ .

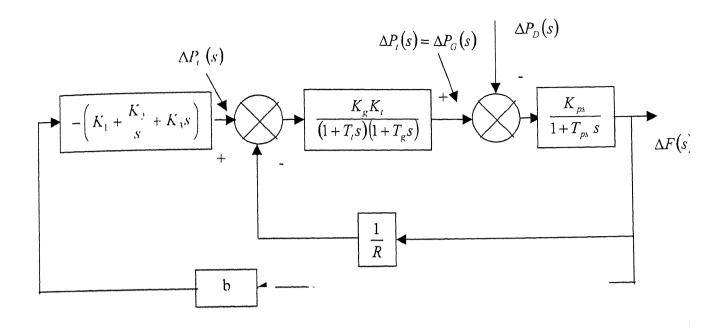


Fig. 3.5 Load Frequency Control with PID controller

The signal  $\Delta P_{C}(s)$  generated by the PID controller must be opposite sign to  $\Delta F(s)$  which accounts for negative sign in the block for integral controller. Now

$$\Delta F(s) = -\frac{RK_{ps}s(1 + T_{g}s)(1 + T_{t}s)}{s(1 + T_{g}s)(1 + T_{t}s)(1 + T_{ps}s)R + K_{ps}\left(\left(K_{1} + \frac{K_{2}}{s} + K_{3}s\right)R + s\right)} \frac{\Delta P_{D}}{s}$$

$$\Delta f \mid_{\text{Steady state}} = \underbrace{Lt}_{s \to 0} s \Delta F(s) = 0 \qquad .....(3.9)$$

# 3.3 GENERATION PRICING

With in the distributed system load is distributed throughout the system and generators are located at specific buses. To simulate the dynamic behavior of the system, disturbances are specified as load fluctuations of small magnitude to allow the use of small-signal, liner models. A system model is defined by specifying the distribution system topology, the location and size of loads and the location, size and type of the generators. The inputs to the models are the system disturbances, represented as the input

vector to the system of state equations. The output from the simulation is the dynamic behavior of all the state variables, with frequency and real power output typically being of greater interest than the others. Monitoring the frequency at each bus can assess the frequency stability.

The goals in developing models for analyzing frequency behavior are to represent the dynamics of distributed generators in response to system disturbances, and to propose and analyze the effectiveness of different control strategies designed to ensure system stability.

The modeling effort is based on building decoupled, linearized state space models for each distributed generator and coupling them through a distribution system model. The models that include a synchronous generator all use a form of the swing equation [1] as the generator state equation:

$$J\ddot{\delta} + D\dot{\delta} = P_m - P_e \qquad ....(3.10)$$

Where  $P_c \equiv P_{cr}$ , the electrical power output.

 $P_m$  is the mechanical power from the turbine.

The turbines in the system are assumed as steam turbines to continue the following analysis. In developing the model of the generator, the objective is to represent each generator with a small number of state variables.

The small signal dynamic for each generator [1] is

$$M w_{G} = (e_{t} - D) w_{G} + P_{t} - P_{G}$$

$$T_{u} P_{t} = -P_{t} + k_{t} a$$

$$T_{g} a = -w_{G} - r a$$
(3.11)

Where

M is the inertia constant

 $e_i$  is a coefficient representing the turbine self-regulation defined as  $\partial P_i/\partial w_G$ 

D is the damping coefficient

 $T_u$  is the time constant representing the delay between the control valves and the turbine nozzles

 $k_{\rm r}$  is a proportionality factor representing the control valve position relative to the turbine output variation

 $T_g$  is the time constant of the valve-servomotor-turbine gate system

a is the gate position

r is the permanent speed droop of the turbine

After getting the deviations in the frequency and generation in each control area following the load changes in every area, the generations cost in every control area are calculated as follows.

The price model presented is for decoupled real power/frequency dynamics. The reason for this is two fold. First, with this emphasis, the modeling effort mirrors the pattern to date for developing a spot price or responsive price system, which usually focuses on pricing real power, since that is the major commodity of the industry. The second reason is that the use of distributed generators for voltage support is reasonably well accepted by the power industry. In contrast, the frequency dynamics of distribution systems with distributed generation units, and the possibility of these units participating in the supply of ancillary services such as frequency stability and spinning reserve, are relatively new issues. Small signal, linearized models are used for analyzing these markets.

The development of the price model begins here by expressing the cost of power generation in terms of the state variables in the generator equations. Cost can be incorporated into the state space generator models by writing an output equation to capture the variable costs associated with generating power from any given technology. Each state space model identifies the set of elements that together can reproduce the basic machine performance. The cost equation [1] then will become now,

$$\cos t = C = C_w w_G + C_p P_t + C_a a + C(P_G)$$
 .....(4.12)

where  $C(P_G)$  is given by the Eq. (2.8)

The coefficients in the above equation represent the marginal cost associated with each piece of equipment or process represented by the specified state variable.

With the addition of the output cost equation, the model for each generator can be expressed as

$$M w_{G} = (e_{t} - D) w_{G} + P_{t} - P_{G}$$

$$T_{u} P_{t} = -P_{t} + k_{t} a$$

$$T_{g} a = -w_{G} - r a + w_{ref} \qquad ............(3.13)$$

$$\cos t = C = C_{w} w_{G} + C_{p} P_{t} + C_{a} a + C(P_{G})$$

The above model can be represented as

$$x = f(x, P_G)$$

$$C = h(x, P_G)$$
.....(3.14)

The generators and the system will respond to the price signal at specific intervals, indicating that the price signal is best modeled in discrete time. The first step in developing this discrete time model is to assume the primary dynamics have settled. Therefore the Eq. (3.14) will now become

$$0 = f(x, P_G) C = h(x, P_G)$$
 .....(3.15)

Solving these equations for cost results in a discrete time cost equation of the form

$$C(k) = \gamma_1 w_G(k) + C(P_G(k)) + \gamma_2 w_{ref}$$
 .....(3.16)

Where k represents the discrete time index for the price control loop, and  $\gamma_1$  and  $\gamma_2$  are constant expressions of the generator parameters and cost coefficients. For the steam generator, these coefficients are of the from

$$\gamma_{1} = \left(C_{w} - \frac{C_{p} k_{t}}{r} - \frac{C_{a}}{r}\right)$$

$$\gamma_{2} = \left(\frac{C_{p} k_{t}}{r} - \frac{C_{a}}{r}\right)$$
.....(3.17)

#### 3.4 RESULTS

All the generators have been assumed to be in a single control area. It is assumed that all the generators are having similar characteristics such that these generators can be taken a single equivalent generator of the same type. The marginal costs are taken as shown in the Table A.7. As the real system data is not available the marginal costs associated with each piece of equipment have been assumed.

Change in load at any bus will ultimately increase load on generators, which will cause frequency deviation at the generator. Current price model has been used to calculate the generation cost considering both the deviation in frequency and the change in real power output. Various test systems have been taken for the study. Data for these systems is shown in appendix A. For a step load change at one of the buses the load flow has been run to find out the actual load change that will occur at the generator, this change has been taken as  $\Delta P_D(s)$  and frequency deviation and generation change have been calculated using the method explained in section 4.2. Using these quantities generation pricing has been done as explained in section 4.3. Results are shown below.

## 3.4.1 Generation Pricing for 43 bus test system

43 bus test system shown in Fig. (A.1) is taken for the study. The data is given in the appendix  $\Lambda$ . The generator model parameters [1] are shown in Table. A.8.

The load at the bus 3 has been increased by 0.05 p.u. The load flow program is again run and the actual load burden on the generators have been calculated which has been taken as  $\Delta P_D$ .

By using Controller this frequency deviation can be brought back to zero, here they have not been used. The main reason for not using the controllers is the users that are drawing excessive load from the scheduled power themselves will reduce the usage of the power as the price is increasing because of the deviations in real power output and frequency. As the demand is more than the supply the frequency settled down at a lower value. The final frequency, generation and price are tabulated below before disturbance,

after disturbance and their peak values in table 4.1. The deviation in frequency is shown in Fig. 3.6. The change in generation is shown in Fig. 3.7.

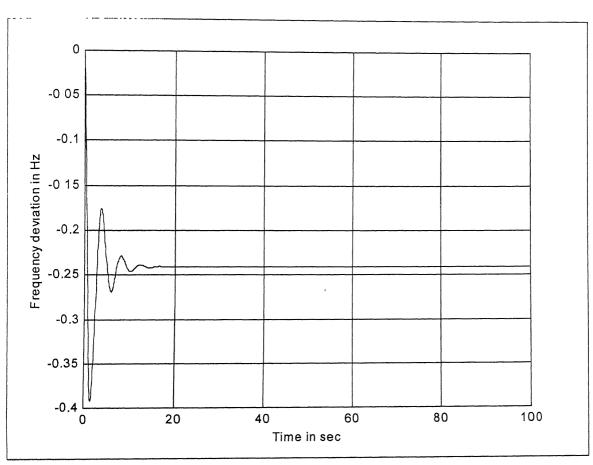


Fig. 3.6 Frequency Deviation in 43 bus power system after a load change of 0.05 p.u at bus 3

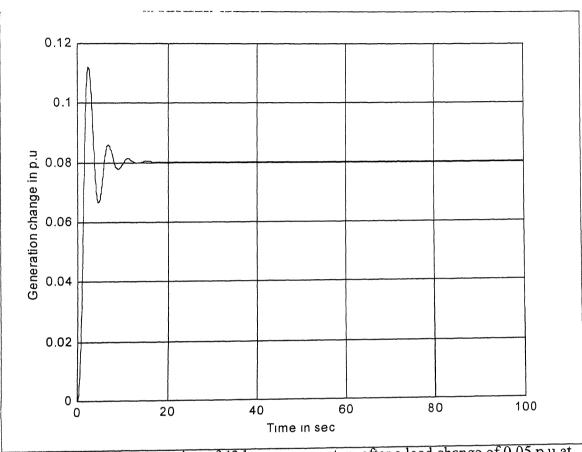


Fig. 3.7 Change in generation of 43 bus power system after a load change of 0.05 p.u at bus3

After getting the deviations in frequency and real power generation the actual frequency and generation have been calculated. The base frequency is taken as 50 Hz. The frequency plot is shown in Fig. 3.8 and the total generation is shown in Fig. 3.9.

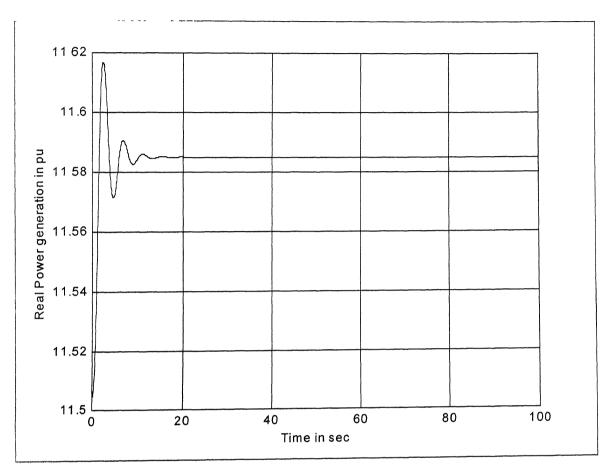


Fig. 3.8 Total generation in 43 bus power system after a load change of 0.05 p.u at bus 3

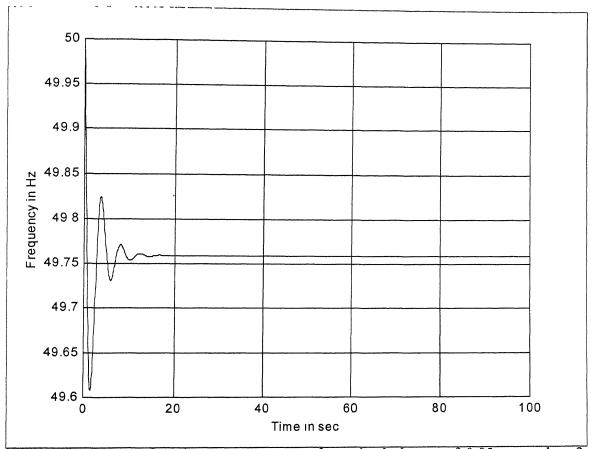


Fig. 3.9 Frequency of 43 bus power system after a load change of 0.05 p.u at bus 3

The generation cost is calculated using the price model after getting the deviations in the frequency and the real power output. The variation in the cost is shown in Fig.3.10.

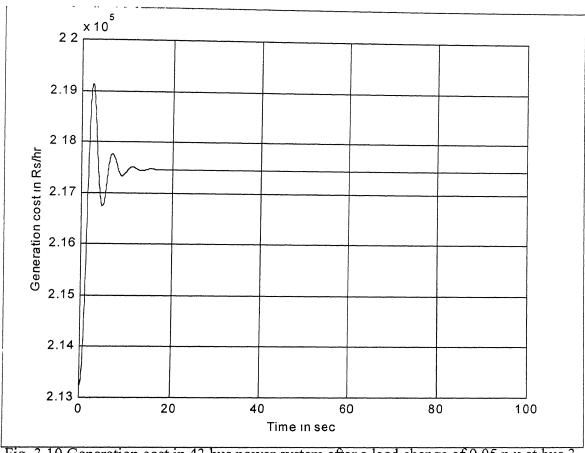


Fig. 3.10 Generation cost in 43 bus power system after a load change of 0.05 p.u at bus 3

Table 3.1 Comparison of the frequency, real power output and generation cost before and after disturbances for 43 bus test system

Quantity	Before disturbance	After disturbance	Peak value
Frequency (Hz)	50	49.7591	49.6073
Generation (p.u.)	11.5047	11.585	11.6167
Price (Rs/hr.)	2.1143*10 <sup>5</sup>	2.1746*10 <sup>5</sup>	2.1913*10 <sup>5</sup>

# 3.4.2 Generation Pricing in 13 bus test system

13 bus test system [4] shown in Fig. (A.2) is taken for the study. The data is given in the appendix A. The generator model parameters [1] are shown in Table. (A.8).

The load at the bus 12 has been increased by 0.02 p.u. The load flow program is again run and the actual load burden on the generators have been calculated which has been taken as  $\Delta P_D$ .

By using Controller this frequency deviation can be brought back to zero, but here they have not been used. The main reason for not using the controllers is the users that are drawing excessive load from the scheduled power themselves will reduce the usage of the power as the price is increasing because of the deviations in real power output and frequency. As the demand is more than the supply the frequency settled down at a lower value. The final frequency, generation and price are tabulated below before disturbance, after disturbance and their peak values in table 4.2. The deviation in frequency is shown in Fig. 3.11. The change in generation is shown in Fig. 3.12.

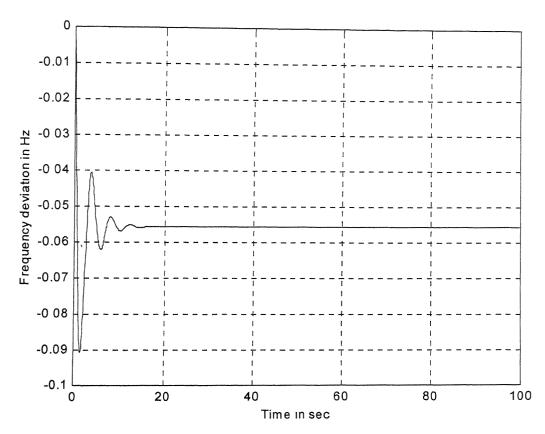


Fig. 3.11 Frequency Deviation in 13 bus power system after a load change of 0.02 p.u at bus 12

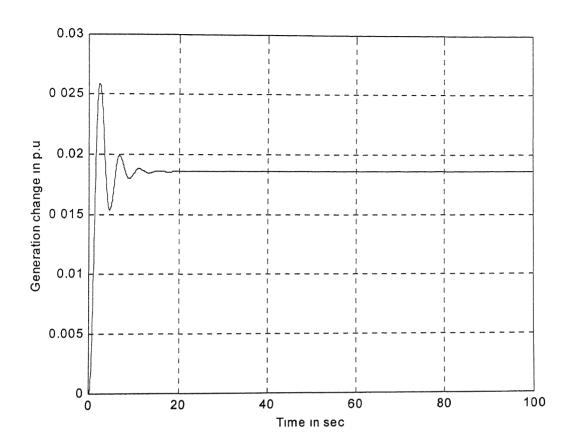


Fig. 3.12 Change in Generation of 13 bus power system after a load change of 0.02 p.u at bus 12

केन्द्रीत पुरतका**लय** भारत्रीय कि कान**पुर**  After getting the deviations in frequency and real power generation the actual frequency and generation have been calculated. The base frequency is taken as 50 Hz. The frequency plot is shown in Fig. 3.13 and the total generation is shown in Fig. 3.14.

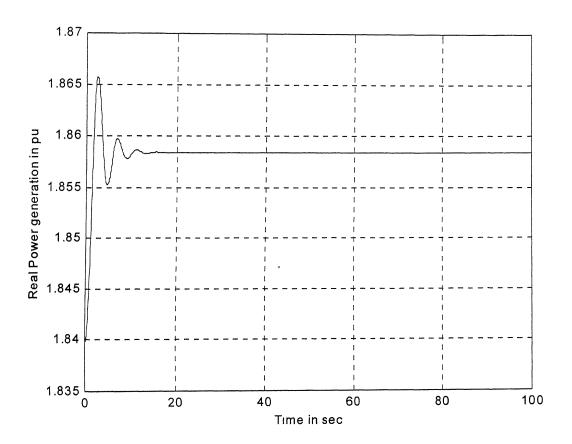


Fig. 3.13 Total generation in 13 bus power system after a load change of 0.02 p.u at bus 12

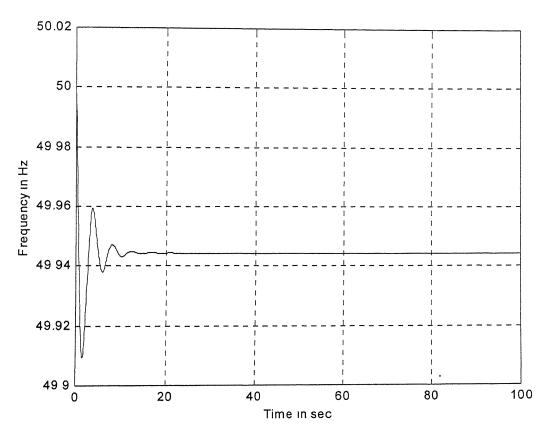


Fig. 3.14 Frequency of 13 bus power system after a load change of 0.02 p.u at bus 12

The generation cost is calculated using the price model after getting the deviations in the frequency and the real power output. The variation in the cost is shown in Fig.3.15.

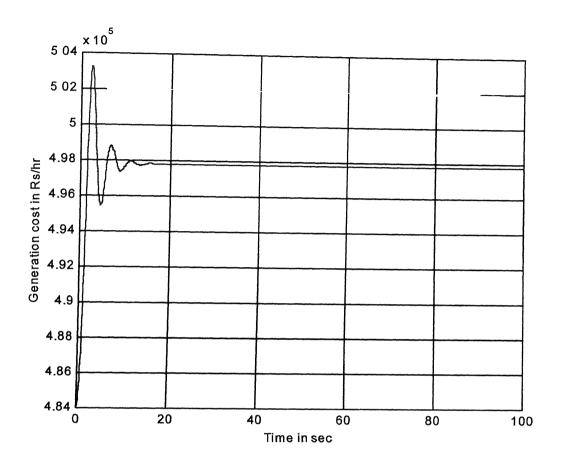


Fig. 3.15 Generation cost in 13 bus power system after a load change of 0.02 p.u at bus 12

Table 3.2 Comparison of the frequency, real power output and generation cost before and after disturbances for 13 bus test system

Quantity	Before disturbance	After disturbance	Peak value
		(steady state)	
Frequency (Hz)	50	49.9443	49.9092
Generation (p.u.)	1.8	1.8584	1.8657
Price (Rs/hr.)	1.5543*10 <sup>5</sup>	1.5557*10 <sup>5</sup>	1.5575*10 <sup>5</sup>

## 3.4.3 Generation Pricing in 14 bus test system

14 bus test system shown in Fig. (A.3) is taken for the study. The data is given in the appendix A. The generator model parameters [1] are shown in Table. (A.8).

The load at the bus 9 has been increased by 0.05 p.u. The load flow program is again run and the actual load burden on the generators have been calculated which has been taken as  $\Delta P_D$ .

By using Controller this frequency deviation can be brought back to zero, but here they have not been used. The main reason for not using the controllers is the users that are drawing excessive load from the scheduled power themselves will reduce the usage of the power as the price is increasing because of the deviations in real power output and frequency. As the demand is more than the supply the frequency settled down at a lower value. The final frequency, generation and price are tabulated below before disturbance, after disturbance and their peak values in table 4.3. The deviation in frequency is shown in Fig. 3.16. The change in generation is shown in Fig. 3.17.

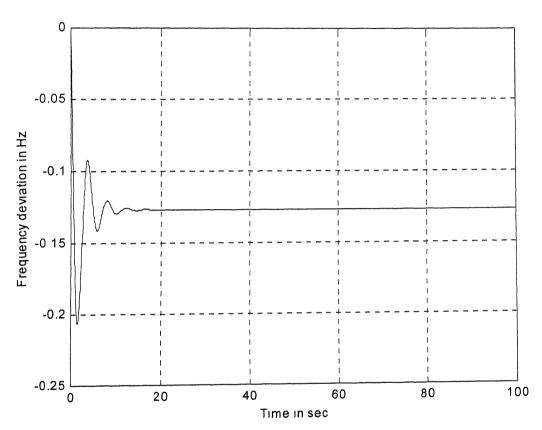


Fig. 3.16 Frequency Deviation in 14 bus power system after a load change of 0.05 p.u at bus 9

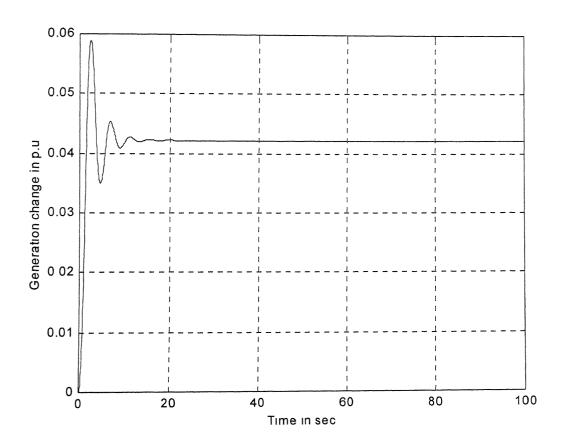


Fig. 3.17 Change in Generation of 14 bus power system after a load change of 0.05 p.u at bus 9

After getting the deviations in frequency and real power generation the actual frequency and generation have been calculated. The base frequency is taken as 50 Hz. The frequency plot is shown in Fig. 3.18 and the total generation is shown in Fig. 3.19.

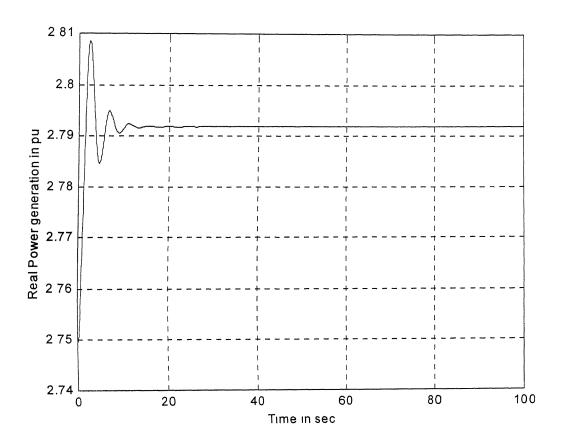


Fig. 3.18 Total generation in 14 bus power system after a load change of 0.05 p.u at bus 9

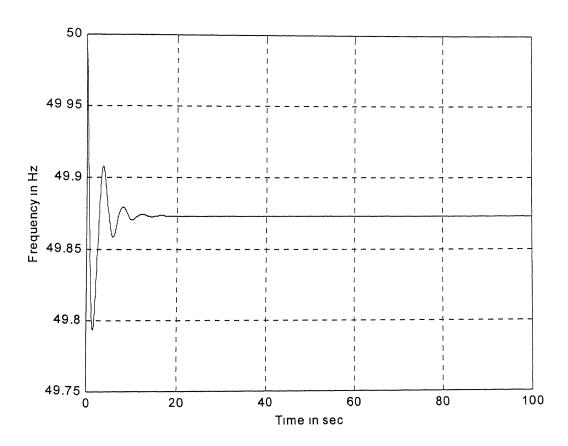


Fig. 3.19 Frequency of 14 bus power system after a load change of 0.05 p.u at bus 9

The generation cost is calculated using the price model after getting the deviations in the frequency and the real power output. The variation in the cost is shown in Fig.3.20.

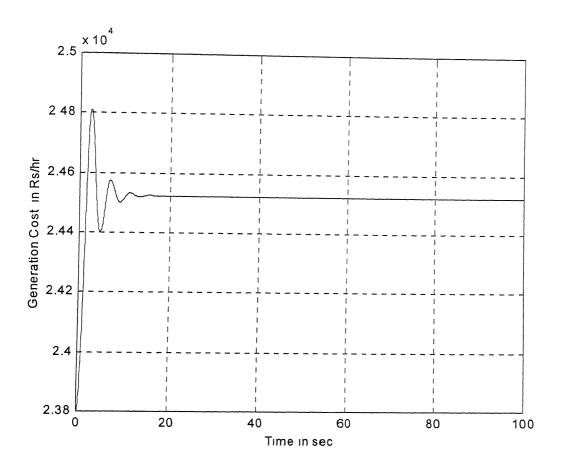


Fig. 3.20 Generation cost in 14 bus power system after a load change of 0.05 p.u at bus 9

Table 3.3 Comparison of the frequency, real power output and generation cost before and after disturbances for 14 bus test system

Quantity	Before disturbance	After disturbance	Peak value
Frequency (Hz)	50	49.8695	49.7873
Generation (p.u.)	2.74	2.7944	2.8116
Price (Rs/hr.)	2.1708*10 <sup>4</sup>	2.4545*10 <sup>4</sup>	2.4842*104

## 3.4.4 Generation Pricing in 11 bus test system

11 bus test system [4] shown in Fig. (A.4) is taken for the study. The data is given in the appendix A. The generator model parameters [1] are shown in Table. (A.8).

The load at the bus 5 has been increased by 0.05 p.u. The load flow program is again run and the actual load burden on the generators have been calculated which has been taken as  $\Delta P_D$ .

By using Controller this frequency deviation can be brought back to zero, but here they have not been used. The main reason for not using the controllers is the users that are drawing excessive load from the scheduled power themselves will reduce the usage of the power as the price is increasing because of the deviations in real power output and frequency. As the demand is more than the supply the frequency settled down at a lower value. The final frequency, generation and price are tabulated below before disturbance, after disturbance and their peak values in table 4.4. The deviation in frequency is shown in Fig. 3.21. The change in generation is shown in Fig. 3.22.

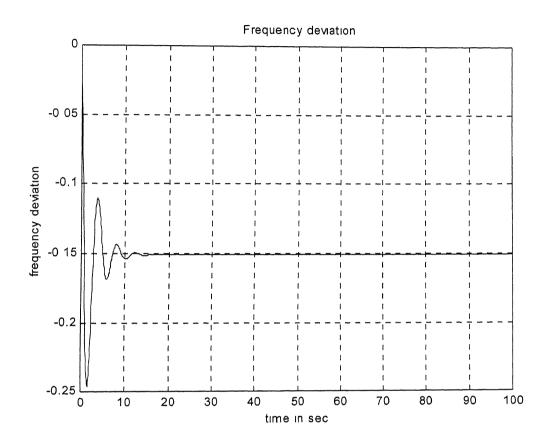


Fig. 3.21 Frequency Deviation in 11 bus power system after a load change of 0.05 p.u at bus 5

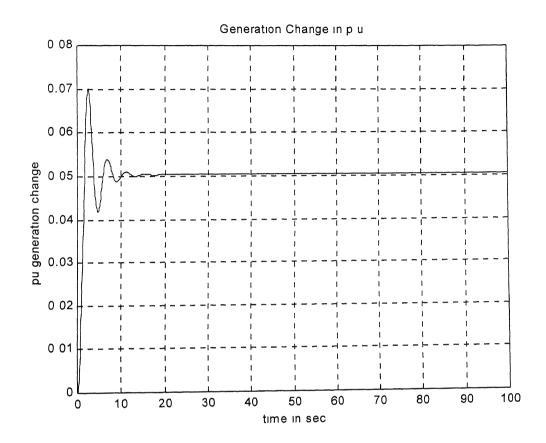


Fig. 3.22 Change in Generation of 11 bus power system after a load change of 0.05 p.u at bus 5

After getting the deviations in frequency and real power generation the actual frequency and generation have been calculated. The base frequency is taken as 50 Hz. The frequency plot is shown in Fig. 3.23 and the total generation is shown in Fig. 3.24.

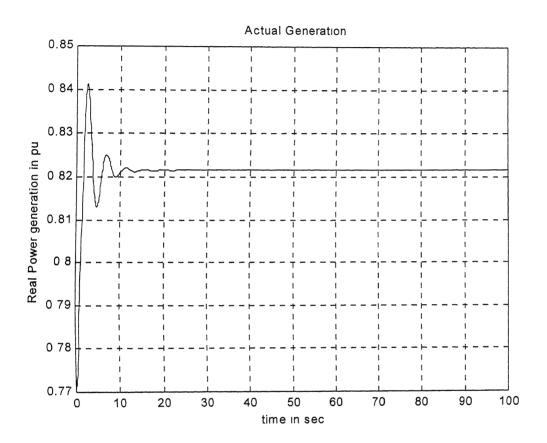


Fig. 3.23 Total generation in 11 bus power system after a load change of 0.05 p.u at bus 5

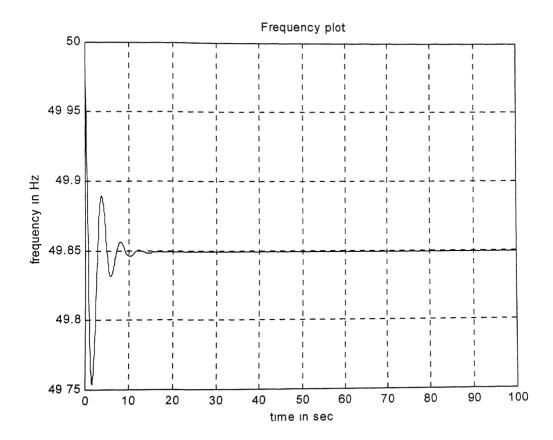


Fig. 3.24 Frequency of 11 bus power system after a load change of 0.05 p.u at bus 5

The generation cost is calculated using the price model after getting the deviations in the frequency and the real power output. The variation in the cost is shown in Fig.3.15.

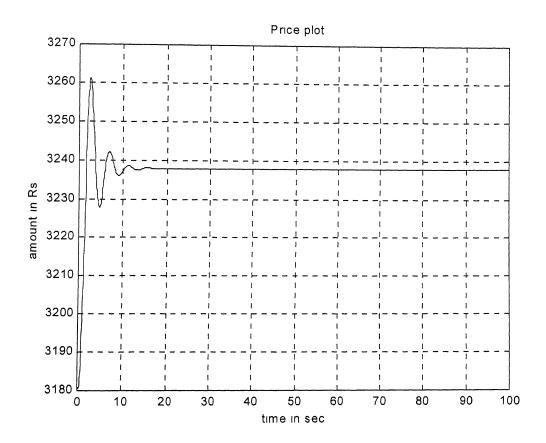


Fig. 3.25 Generation cost in 11 bus power system after a load change of 0.05 p.u at bus 5

Table 3.2 Comparison of the frequency, real power output and generation cost before and after disturbances for 13 bus test system

Quantity	Before disturbance	After disturbance	Peak value
Frequency (Hz)	50	49.849	49.75
Generation (p.u.)	0.7708	0.8215	0.8414
Price (Rs/hr.)	3158	3237	3261

### **Chapter 4**

# CONCLUSIONS AND SCOPE FOR FURTHER WORK

#### 4.1 Conclusions

The load flow problems for various test systems have been solved using NRLF method with and without constraints. The general base case solution for 11 bus system without constraints is not a better solution as the voltage magnitudes are very far from the operating voltage. Where as if the limits on the bus voltages are considered it has converged to a better solution. For 13 bus system also if the constraints are not taken the voltages are higher in magnitude. When the constraints are considered the voltage magnitudes have been limited with in the prescribed limits. For the other systems there is no such difference in the solution even when the constraints are considered.

In Optimal Power flow the objective function has been taken as the minimizing the generator operating cost. For every system the optimal power flow has been run to reschedule the generation, which is obtained from the load flow solution to reduce the generation cost.

All the systems have been considered as single isolated control areas. When the load at any bus changes it will cause deviation in system frequency as well as real power generation. This study has been done on four test systems. In every system the deviations in frequency and real power generation have been calculated following a load change at any bus. The generation cost has been taken as a function of both frequency and real power generation. When the load increases the generation cost will increase and viceversa as the frequency decreases and generation increases. The PID controllers have not been used in the thesis work. The reason is when any user draws more than scheduled power the increase in generation cost will be charged to that particular user who will then shed the load automatically. Then the frequency will settle back to the normal value.

### 4.2 Scope for Further work

- The work can be extended to penalize the users those are causing frequency changes by drawing unscheduled power in such a way this extra charge which they have to pay will make the load to come down and the frequency to go upto the normal value.
- The operating cost can be minimized by taking the constraints on operating frequency also. Where as in the present thesis work only the load flow constraints have been taken.
- In this thesis work every system has been taken as a single control area. One
  can take a system such that it can be divided into a number of control areas
  and then the price model can be extended for calculating generation cost in
  every area following the load changes.

### Data For 43 Bus Test System

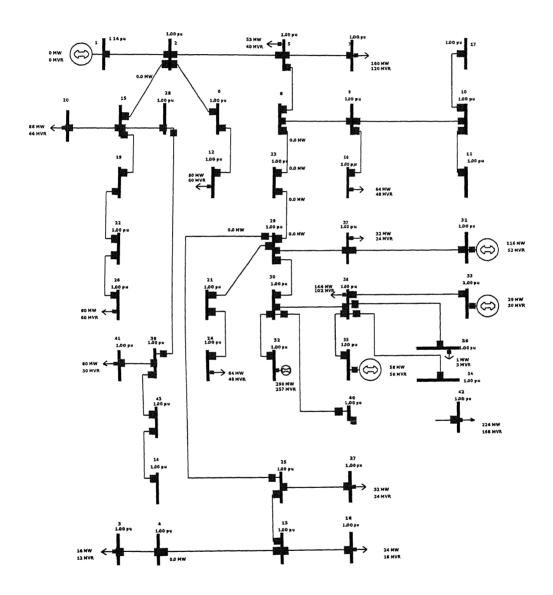


Fig A.1 43 bus test system

The 43 bus system is shown in Fig. A.1. The system data is taken from ref. [4, 7]. The relevant data are provided in following tables. Table A.1 gives Y Bus matrix elements of 43 bus system, Table A.2 gives the operating condition of the system.

Table A.1 Y Bus matrix elements of 43 bus system

From			B <sub>1</sub>	
Node (i)	(j)		-	
1	1	0	-30.609	
1	2	0	30.609	
2	2	481.288	-1545.194	
2	5	-277.195	873.583	
2	6	-34.368	108.124	
2	15	-169.726	534.322	
3	3	0	-5.714	
3	4	0	6.015	
4	4	61.331	-69.160	
4	13	-61.331	62.874	
5	5	277.195	-916.892	
5	7	0	21.277	
5	8	0	20.513	
6	6	34.368	-118.699	
6	12	0	10.638	
7	7	0	-20	
8	8	452.840	-482.861	
8	9	-288.938	295.777	
8	23	-163.902	167.191	
9	9	300.983	-317.044	
9	10	-12.045	12.342	
9	16	0.0	8.796	
10	10	12.045	-20.885	
10	11	0.0	2.857	
10	17	0.0	5.714	
11	11	0.0	-2.857	
12	12	0.0	-10.0	
13	13	92.381	-100.709	
13	18	0.0	6.015	
13	25	-31.05	31.640	
14	14	0.0	-15.015	
14	43	0.0	15.4	
15	15	340.398	-916.783	
15	19	0.0	8.649	
15	20	0.0	15.791	
15	28	-170.673	357.003	
16	16	0.0	-8.576	
17	17	0.0	-5.714	
18	18	0	-5.714	
19	19	164.292	-280.783	

19				
21         21         104.312         -143.609           21         24         0         9.267           21         29         -104.312         133.623           22         22         164.292         -282.281           22         26         0         9.023           23         23         321.579         -328.81           23         29         -157.677         161.76           24         24         0         -8.572           25         25         87.15         -106.814           25         27         0         9.023           25         27         0         9.023           25         27         0         9.023           25         27         0         9.023           25         27         0         -8.572           27         27         0         -8.572           28         28         373.447         -612.837           28         39         -202.775         256.136           29         29         318.089         -372.311           29         30         0         3.766           29         37 </td <td>19</td> <td>22</td> <td>-164.292</td> <td>272.805</td>	19	22	-164.292	272.805
21         24         0         9.267           21         29         -104.312         133.623           22         22         164.292         -282.281           23         23         321.579         -328.81           23         29         -157.677         161.76           24         24         0         -8.572           25         25         87.15         -106.814           25         27         0         9.023           25         29         -56.1         65.824           26         26         0         -8.572           27         27         0         -8.572           28         28         373.447         -612.837           28         39         -202.775         256.136           29         29         318.089         -372.311           29         30         0         3.766           29         37         0         7.895           30         30         125.789         -524.464           30         32         0         30.769           30         38         0         4.131           30 <t< td=""><td></td><td></td><td></td><td>-15.002</td></t<>				-15.002
21         29         -104.312         133.623           22         22         164.292         -282.281           22         26         0         9.023           23         23         321.579         -328.81           23         29         -157.677         161.76           24         24         0         -8.572           25         25         87.15         -106.814           25         27         0         9.023           25         29         -56.1         65.824           26         26         0         -8.572           27         27         0         -8.572           27         27         0         -8.572           28         39         -202.775         256.136           29         29         318.089         -372.311           29         30         0         3.766           29         37         0         7.895           30         30         125.789         -524.464           30         32         0         30.769           30         32         0         30.769           30         38 <td></td> <td></td> <td>104.312</td> <td></td>			104.312	
22         22         164.292         -282.281           22         26         0         9.023           23         23         321.579         -328.81           23         29         -157.677         161.76           24         24         0         -8.572           25         25         87.15         -106.814           25         27         0         9.023           25         29         -56.1         65.824           26         26         0         -8.572           27         27         0         -8.572           28         28         373.447         -612.837           28         28         373.447         -612.837           28         29         29         318.089         -372.311           29         29         318.089         -372.311           29         30         0         3.766           29         37         0         7.895           30         30         125.789         -524.464           30         32         0         30.769           30         32         0         30.769           <				9.267
22         26         0         9.023           23         23         321.579         -328.81           23         29         -157.677         161.76           24         24         0         -8.572           25         25         25         87.15         -106.814           25         27         0         9.023           25         29         -56.1         65.824           26         26         0         -8.572           27         27         0         -8.572           28         28         373.447         -612.837           28         39         -202.775         256.136           29         29         318.089         -372.311           29         30         0         3.766           29         37         0         7.895           30         30         125.789         -524.464           30         32         0         30.769           30         38         0         4.131           30         40         -125.789         485.547           31         31         0         -13.038           32 <td></td> <td></td> <td>-104.312</td> <td>133.623</td>			-104.312	133.623
23         23         321.579         -328.81           23         29         -157.677         161.76           24         24         0         -8.572           25         25         87.15         -106.814           25         27         0         9.023           25         29         -56.1         65.824           26         26         0         -8.572           27         27         0         -8.572           28         28         373.447         -612.837           28         39         -202.775         256.136           29         29         318.089         -372.311           29         30         0         3.766           29         37         0         7.895           30         30         125.789         -524.464           30         32         0         30.769           30         38         0         4.131           30         40         -125.789         485.547           31         31         37         0         13.038           31         37         0         13.038           32 <td></td> <td></td> <td></td> <td>-282.281</td>				-282.281
23         29         -157.677         161.76           24         24         0         -8.572           25         25         87.15         -106.814           25         27         0         9.023           25         29         -56.1         65.824           26         26         0         -8.572           27         27         0         -8.572           28         28         373.447         -612.837           28         39         -202.775         256.136           29         29         318.089         -372.311           29         30         0         3.766           29         37         0         7.895           30         30         125.789         -524.464           30         32         0         30.769           30         38         0         4.131           30         40         -125.789         485.547           31         31         31         0         -13.038           31         31         0         -13.038           32         32         0         -30.769           33				9.023
24         24         0         -8.572           25         25         87.15         -106.814           25         27         0         9.023           25         29         -56.1         65.824           26         26         0         -8.572           27         27         0         -8.572           28         28         373.447         -612.837           28         39         -202.775         256.136           29         29         318.089         -372.311           29         30         0         3.766           29         37         0         7.895           30         30         32.769         -524.464           30         32         0         30.769           30         38         0         4.131           30         40         -125.789         485.547           31         31         30         -13.038           31         31         0         -13.038           32         32         0         -30.769           33         33         33         0         -3.320           33			321.579	-328.81
25         25         87.15         -106.814           25         27         0         9.023           25         29         -56.1         65.824           26         26         0         -8.572           27         27         0         -8.572           28         28         373.447         -612.837           28         39         -202.775         256.136           29         29         318.089         -372.311           29         30         0         3.766           29         37         0         7.895           30         30         125.789         -524.464           30         32         0         30.769           30         38         0         4.131           30         38         0         4.131           30         40         -125.789         485.547           31         31         31         0         -13.038           32         32         0         -30.769           33         33         33         0         -3.320           33         33         38         0         -3.320		29	-157.677	161.76
25         27         0         9.023           25         29         -56.1         65.824           26         26         0         -8.572           27         27         0         -8.572           28         28         373.447         -612.837           28         39         -202.775         256.136           29         29         318.089         -372.311           29         30         0         3.766           29         37         0         7.895           30         30         125.789         -524.464           30         32         0         30.769           30         32         0         30.769           30         38         0         4.131           30         40         -125.789         485.547           31         31         31         0         -13.038           31         31         37         0         13.038           32         32         0         -30.769           33         33         33         0         -3.320           33         33         38         0         -7.365		24	0	-8.572
25         29         -56.1         65.824           26         26         0         -8.572           27         27         0         -8.572           28         28         373.447         -612.837           28         39         -202.775         256.136           29         29         318.089         -372.311           29         30         0         3.766           29         37         0         7.895           30         30         125.789         -524.464           30         32         0         30.769           30         38         0         4.131           30         40         -125.789         485.547           31         31         0         -13.038           31         37         0         13.038           32         32         0         -30.769           33         33         33         0         -3.320           33         33         33         0         -3.320           33         33         33         0         -7.365           34         38         0         6.852		25	87.15	-106.814
26         26         0         -8.572           27         27         0         -8.572           28         28         373.447         -612.837           28         39         -202.775         256.136           29         29         318.089         -372.311           29         30         0         3.766           29         37         0         7.895           30         30         125.789         -524.464           30         32         0         30.769           30         38         0         4.131           30         40         -125.789         485.547           31         31         0         -13.038           31         37         0         13.038           32         32         0         -30.769           33         33         0         -3.320           33         33         0         -3.320           33         33         0         -3.320           34         34         0         -7.365           34         38         0         6.18           35         38         0	25	27	0	9.023
27         27         0         -8.572           28         28         373.447         -612.837           28         39         -202.775         256.136           29         29         318.089         -372.311           29         30         0         3.766           29         37         0         7.895           30         30         125.789         -524.464           30         32         0         30.769           30         38         0         4.131           30         40         -125.789         485.547           31         31         0         -13.038           31         31         0         -13.038           32         32         0         -30.769           33         33         0         -3.320           33         33         0         -3.320           33         33         0         -3.320           34         34         0         -7.365           34         38         0         6.18           35         35         0         -6.18           35         38         0	25	29	-56.1	65.824
28         28         373.447         -612.837           28         39         -202.775         256.136           29         29         318.089         -372.311           29         30         0         3.766           29         37         0         7.895           30         30         125.789         -524.464           30         32         0         30.769           30         38         0         4.131           30         40         -125.789         485.547           31         31         0         -13.038           31         31         0         -13.038           32         32         0         -30.769           33         33         33         0         -3.320           33         33         33         0         -3.320           33         38         0         3.32           34         34         34         0         -7.365           34         38         0         6.852           35         35         0         -6.18           35         38         0         2.703	26	26	0	-8.572
28       39       -202.775       256.136         29       29       318.089       -372.311         29       30       0       3.766         29       37       0       7.895         30       30       125.789       -524.464         30       32       0       30.769         30       38       0       4.131         30       40       -125.789       485.547         31       31       0       -13.038         31       31       0       -13.038         32       32       0       -30.769         33       33       30       -3.320         33       33       30       -3.320         33       33       30       -3.320         33       34       34       0       -7.365         34       38       0       6.852         35       35       35       0       -6.18         35       38       0       2.703         36       38       0       2.703         37       37       0       -21.348         38       38       0       -22.398	27			-8.572
29       29       318.089       -372.311         29       30       0       3.766         29       37       0       7.895         30       30       125.789       -524.464         30       32       0       30.769         30       38       0       4.131         30       40       -125.789       485.547         31       31       0       -13.038         31       37       0       13.038         32       32       0       -30.769         33       33       0       -3.320         33       33       0       -3.320         33       33       0       -3.320         33       33       0       -3.320         34       34       0       -7.365         34       38       0       6.852         35       35       0       -6.18         35       38       0       2.703         36       38       0       2.703         37       37       0       -21.348         38       38       0       -22.398         39       41	28	28	373.447	-612.837
29       30       0       3.766         29       37       0       7.895         30       30       125.789       -524.464         30       32       0       30.769         30       38       0       4.131         30       40       -125.789       485.547         31       31       0       -13.038         31       37       0       13.038         32       32       0       -30.769         33       33       0       -3.320         33       33       0       -3.320         33       38       0       3.32         34       34       0       -7.365         34       38       0       6.852         35       35       0       -6.18         35       38       0       2.703         36       38       0       2.703         37       37       0       -21.348         38       38       0       -22.398         39       41       0       15.015         39       41       0       15.015         40       40	28	39	-202.775	256.136
29       37       0       7.895         30       30       125.789       -524.464         30       32       0       30.769         30       38       0       4.131         30       40       -125.789       485.547         31       31       0       -13.038         31       37       0       13.038         32       32       0       -30.769         33       33       30       -3.320         33       38       0       -3.320         33       38       0       -7.365         34       34       0       -7.365         34       38       0       6.852         35       35       0       -6.18         35       38       0       -2.703         36       38       0       -2.703         36       38       0       -21.348         38       38       0       -22.398         39       39       512.581       -663.260         39       41       0       15.015         39       43       -309.806       392.255         40	29	29	318.089	-372.311
30       30       125.789       -524.464         30       32       0       30.769         30       38       0       4.131         30       40       -125.789       485.547         31       31       0       -13.038         31       37       0       13.038         32       32       0       -30.769         33       33       0       -3.320         33       38       0       3.32         34       34       0       -7.365         34       38       0       6.852         35       35       0       -6.18         35       38       0       6.18         36       36       0       -2.703         36       38       0       2.703         37       37       0       -21.348         38       38       0       -22.398         39       39       512.581       -663.260         39       41       0       15.015         39       43       -309.806       392.255         40       40       125.789       -508.837         40 <td>29</td> <td>30</td> <td>0</td> <td>3.766</td>	29	30	0	3.766
30       32       0       30.769         30       38       0       4.131         30       40       -125.789       485.547         31       31       0       -13.038         31       37       0       13.038         32       32       0       -30.769         33       33       0       -3.320         33       38       0       -3.320         33       38       0       -7.365         34       34       0       -7.365         34       38       0       6.18         35       35       35       0       -6.18         35       38       0       6.18         36       38       0       2.703         36       38       0       -21.348         38       38       0       -22.398         39       39       512.581       -663.260         39       41       0       15.015         39       43       -309.806       392.255         40       40       125.789       -508.837         40       42       0       21.622	29	37		7.895
30       38       0       4.131         30       40       -125.789       485.547         31       31       0       -13.038         31       37       0       13.038         32       32       0       -30.769         33       33       0       -3.320         33       38       0       3.32         34       34       0       -7.365         34       38       0       6.852         35       35       0       -6.18         35       38       0       -2.703         36       38       0       2.703         37       37       0       -21.348         38       38       0       -22.398         39       39       512.581       -663.260         39       41       0       15.015         39       43       -309.806       392.255         40       40       125.789       -508.837         40       42       0       21.622         41       41       0       -15.015         42       42       0       -20	30	30	125.789	
30     40     -125.789     485.547       31     31     0     -13.038       31     37     0     13.038       32     32     0     -30.769       33     33     0     -3.320       33     38     0     3.32       34     34     0     -7.365       34     38     0     6.852       35     35     0     -6.18       35     38     0     6.18       36     36     0     -2.703       36     38     0     2.703       37     37     0     -21.348       38     38     0     -22.398       39     41     0     15.015       39     43     -309.806     392.255       40     40     125.789     -508.837       40     42     0     21.622       41     41     0     -15.015       42     42     0     -20	30	32	0	30.769
31       31       0       -13.038         31       37       0       13.038         32       32       0       -30.769         33       33       0       -3.320         33       38       0       3.32         34       34       0       -7.365         34       38       0       6.852         35       35       0       -6.18         35       38       0       6.18         36       36       0       -2.703         36       38       0       2.703         37       37       0       -21.348         38       38       0       -22.398         39       39       512.581       -663.260         39       41       0       15.015         39       43       -309.806       392.255         40       40       125.789       -508.837         40       42       0       21.622         41       41       0       -15.015         42       42       0       -20	30	38		
31     37     0     13.038       32     32     0     -30.769       33     33     0     -3.320       33     38     0     3.32       34     34     0     -7.365       34     38     0     6.852       35     35     0     -6.18       35     38     0     6.18       36     36     0     -2.703       36     38     0     2.703       37     0     -21.348       38     38     0     -22.398       39     39     512.581     -663.260       39     41     0     15.015       39     43     -309.806     392.255       40     40     125.789     -508.837       40     42     0     21.622       41     41     0     -15.015       42     42     0     -20	30	40	-125.789	
32     32     0     -30.769       33     33     0     -3.320       33     38     0     3.32       34     34     0     -7.365       34     38     0     6.852       35     35     0     -6.18       35     38     0     6.18       36     36     0     -2.703       36     38     0     2.703       37     37     0     -21.348       38     38     0     -22.398       39     39     512.581     -663.260       39     41     0     15.015       39     43     -309.806     392.255       40     40     125.789     -508.837       40     42     0     21.622       41     41     0     -15.015       42     42     0     -20			0	
33     33     0     -3.320       33     38     0     3.32       34     34     0     -7.365       34     38     0     6.852       35     35     0     -6.18       35     38     0     6.18       36     36     0     -2.703       36     38     0     2.703       37     0     -21.348       38     38     0     -22.398       39     39     512.581     -663.260       39     41     0     15.015       39     43     -309.806     392.255       40     40     125.789     -508.837       40     42     0     21.622       41     41     0     -15.015       42     42     0     -20				
33     38     0     3.32       34     34     0     -7.365       34     38     0     6.852       35     35     0     -6.18       35     38     0     6.18       36     36     0     -2.703       36     38     0     2.703       37     37     0     -21.348       38     38     0     -22.398       39     39     512.581     -663.260       39     41     0     15.015       39     43     -309.806     392.255       40     40     125.789     -508.837       40     42     0     21.622       41     41     0     -15.015       42     42     0     -20				
34     34     0     -7.365       34     38     0     6.852       35     35     0     -6.18       35     38     0     6.18       36     36     0     -2.703       36     38     0     2.703       37     37     0     -21.348       38     38     0     -22.398       39     39     512.581     -663.260       39     41     0     15.015       39     43     -309.806     392.255       40     40     125.789     -508.837       40     42     0     21.622       41     41     0     -15.015       42     42     0     -20	33			
34     38     0     6.852       35     35     0     -6.18       35     38     0     6.18       36     36     0     -2.703       36     38     0     2.703       37     37     0     -21.348       38     38     0     -22.398       39     39     512.581     -663.260       39     41     0     15.015       39     43     -309.806     392.255       40     40     125.789     -508.837       40     42     0     21.622       41     41     0     -15.015       42     42     0     -20				
35     35     0     -6.18       35     38     0     6.18       36     36     0     -2.703       36     38     0     2.703       37     37     0     -21.348       38     38     0     -22.398       39     39     512.581     -663.260       39     41     0     15.015       39     43     -309.806     392.255       40     40     125.789     -508.837       40     42     0     21.622       41     41     0     -15.015       42     42     0     -20	34			
35     38     0     6.18       36     36     0     -2.703       36     38     0     2.703       37     37     0     -21.348       38     38     0     -22.398       39     39     512.581     -663.260       39     41     0     15.015       39     43     -309.806     392.255       40     40     125.789     -508.837       40     42     0     21.622       41     41     0     -15.015       42     42     0     -20				
36     36     0     -2.703       36     38     0     2.703       37     37     0     -21.348       38     38     0     -22.398       39     39     512.581     -663.260       39     41     0     15.015       39     43     -309.806     392.255       40     40     125.789     -508.837       40     42     0     21.622       41     41     0     -15.015       42     42     0     -20				
36     38     0     2.703       37     37     0     -21.348       38     38     0     -22.398       39     39     512.581     -663.260       39     41     0     15.015       39     43     -309.806     392.255       40     40     125.789     -508.837       40     42     0     21.622       41     41     0     -15.015       42     42     0     -20				
37     37     0     -21.348       38     38     0     -22.398       39     39     512.581     -663.260       39     41     0     15.015       39     43     -309.806     392.255       40     40     125.789     -508.837       40     42     0     21.622       41     41     0     -15.015       42     42     0     -20				
38     38     0     -22.398       39     39     512.581     -663.260       39     41     0     15.015       39     43     -309.806     392.255       40     40     125.789     -508.837       40     42     0     21.622       41     41     0     -15.015       42     42     0     -20				
39     39     512.581     -663.260       39     41     0     15.015       39     43     -309.806     392.255       40     40     125.789     -508.837       40     42     0     21.622       41     41     0     -15.015       42     42     0     -20				
39     41     0     15.015       39     43     -309.806     392.255       40     40     125.789     -508.837       40     42     0     21.622       41     41     0     -15.015       42     42     0     -20				
39     43     -309.806     392.255       40     40     125.789     -508.837       40     42     0     21.622       41     41     0     -15.015       42     42     0     -20				
40     40     125.789     -508.837       40     42     0     21.622       41     41     0     -15.015       42     42     0     -20				
40     42     0     21.622       41     41     0     -15.015       42     42     0     -20				
41     41     0     -15.015       42     42     0     -20				
42 42 0 -20				
A Gui				
43   43   309.806   -408.029	The second secon			
	43	43	309.806	-408.029

Table A.2 Operating condition of 43 bus system

Bus No.	Voltage magnitude (p.u)	Phase Angle (deg.)	Net Real Power (p.u)	Net Reactive Power (p.u)
1	1.136	0	0	0
2	1	0	0	0
3	1	0	-0.16	-0.12
4	1	0	0	0
5	1	0	-0.53	-0.4
6	1	0	0	0
7	1	0	-1.6	-1.2
8	1	0	0	0
9	1	0	0	0
10	1	0	0	0
11	1	0	0	0
12	1	0	-0.8	-0.6
13	1	0	0	0
14	1	0	-0.8	-0.6
15	1	00	0	0
16	1	0	-0.64	-0.48
17	1	00	0	0
18	1	00	-0.24	-0.18
19	1	00	0	0
20	1	0	-0.88	-0.66
21	1	00	0	0
22	1	0	0	0
23	1	0	0	0
24	1	0	-0.64	-0.48
25	1	0	0	0
26	1	0	-0.8	-0.6
27	1	0	-0.32	-0.24
28	1	0	0	0
29	1	0	0	0
30	1	0	0	0.52
31	1	0	1.16	2.57
32	1	0	0.285	0.3
33	1 1		0.285	0.3
34 35	1 1	0	0.58	0.56
35	1	0	-0.005	0.030
36	1 1	0	0	0.030
38		0	-1.44	-1.02
39		0	0	0
40	1 1	0	0	0
41	1 1	0	-0.8	-0.3
42	1 1	0	-2.24	-1.68
43	$\frac{1}{1}$	0	0	0

## Data For 13 Bus Test System

The 13 bus system is shown in Fig. A.2. The system data is taken from ref. [4, 7]. The relevant data are provided in following tables. Table A.3 gives bus data, Table A.4 gives Line data and Table A.5 gives Transformer data.

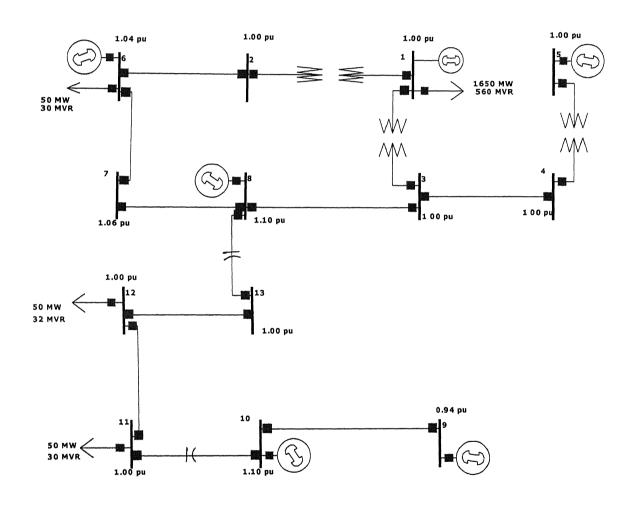


Fig. A.1. 13 Bus power system

Table A.3. Bus data for 13 bus system

Bus	Bus	Phase	Gene	ration	Lo	oad
No.	Voltage	Angle	Real	Reactive	Real	Reactive
	(pu)	(deg.)	Power	Power	(pu)	(pu)
			(pu)	(pu)		
1	1.0*	0.0			1.65	0.56
2	1.0	0.0	0	0	0	0
3	1.0	0.0	0	0	0	0
4	1.0	0.0	0	0	0	0
5	1.0	0.0	0	0	0	0
6	1.037*	0.0	0.5	_	0.05	0.03
7	1.063	0.0	0	0	0	0
8	1.1*	0.0	0	_	0	0
9	0.943*	0.0	0.5	_	0	0
10	1.1*	0.0	0	_	0	0
11	1.0	0.0	0	0	0.05	0.05
12	1.0	0.0	0	0	0.05	0.032
13	1.0	0.0	0	0	0	0

<sup>\*</sup>Input data

Table A.4. Line data for 13 bus ill-conditioned system

Branch	From	To	Resistance	Reactance	Susceptance
Number	Node	Node	(pu)	(pu)	(pu)
1	1	2	0.0040	0.085	0
2	1	3	0.0040	0.0947	0
3	5	4	0.0040	0.0947	0
4	4	3	0.0074	0.1430	0.436
5	6	2	0.0481	0.4590	0.246
6	6	7	0.0090	0.1080	0.016
7	8	3	0.0121	0.233	0.712
8	7	8	0	0.15	0
9	9	10	0.0105	0.2020	0.620
10	10	11	0	-0.15	0
11	11	12	0.0086	0.1665	0.508
12	12	13	0.0075	0.1465	0.448
13	13	8	0	-0.15	0

Base=1000 MVA

Table A.5. Transformer data for 13 bus system

Branch number	From node	To node	Tap setting
1	1	2	+ 5%
2	2	3	+ 10%
3	5	4	+ 10%

Table A.6 Assumed Values for the Cost Coefficients

Coefficient	Value
a	1.4
Ъ	0.8
d	10

Table A.7 Marginal costs associated with various equipments

Coefficient	Value
$C_p$	175
$C_{w}$	200
$C_a$	175

Table A.8 Generator Model Parameters

Parameter	Value
M	1.26
D	2.0
e <sub>t</sub>	0.15
$T_{\rm u}$	0.2
k <sub>t</sub>	0.95
$T_{g}$	0.25
R	2.4

### **DATA FOR 14 BUS SYSTEM**

The 14 bus system is shown in Fig A.3, data is taken from ref. [9] and buses are renumbered. The relevant data are provided in following tables. Table A.9 gives Generator Data, Table A.10 gives Generator Bus voltage, Table A.11 gives Transformer Data, Table A.12 gives Load Bus data and Table A.13 gives Line Data.

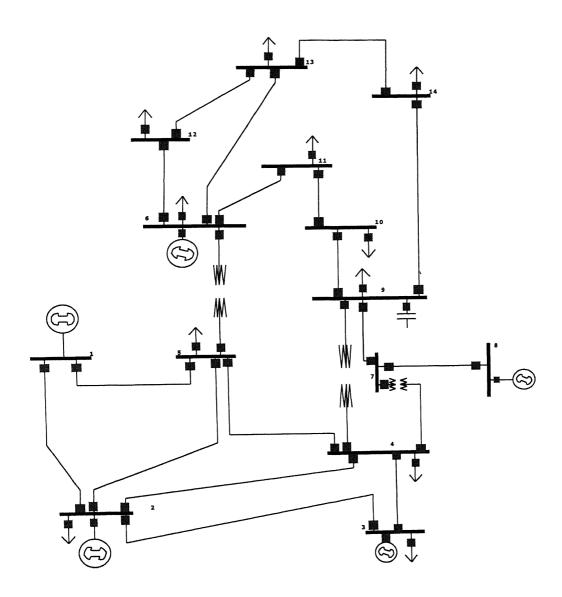


Fig A.3 IEEE 14 bus test system

Table A.9: Generator Data for 14 bus system

Generator	Real Power Generation Limit		Reactive Power Generation Limit	
No	Maximum (MW)	Minimum (MW)	Maximum (MW)	Minimum (MW)
1	200.00	050 0	100.0	-45.0
2	100.0	020.0	0500	-40.0
3	-	-	040.0	0.00
6	150.0	020.0	024.0	-06.0
8	-	-	024.0	-06.0

Table A.10: Generator Bus voltage Data for 14 bus system

Bus	Scheduled Real	Specifieed Voltage	Lo	ad
No.	Power Generation	Magnitude	Real	Reactive
	$P_{G}(MW)$	$V_{spec}(p.u)$	(MW)	(MVAR)
1	_	1.060	00.00	00.00
2	40.0	1.045	21.70	12.70
3	-	1.010	94.20	19.00
6	20.0	1.070	11.2	7.5
8	_	1.090	00.00	00.00

Table A.11: Transformer Data for 14 bus system

Line	From	To	Series Imp	T. C					
No	Bus	Bus	Resistance (0.u)	Reactance (p.u)	Tap Setting				
8	4	7	0.0	0.2091	0.978				
9	4	9	0.0	0.5561	0.969				
10	5	6	0.0	0.2502	0.962				

Table A.12 Load Bus Data for 14 bus system

Bus no	Loa	External Shunt	
Dus no	Real	Reactive	Susceptance (p.u)
4	47.8	4.0	0.0
5	7.6	1.6	0.0
7	0.0	0.0	0.0
9	29.5	16.6	0.19
10	9.0	5.8	0.0
11	3.5	1.8	0.0
12	6.1	1.6	0.0
13	13.5	5.8	0.0
14	14.9	5.0	0.0

Base MVA=100

Table A.13: Line Data for 14 bus system

Line.	From	То	Series Imp	Shunt	
No.			Resistance (p.u)	Reactance (p.u)	Susceptance (p.u)
1	1	2	0.01938	0.05917	0.528
2	1	5	0 05403	0 22304	0.0492
3	2	3	0.04699	0.19797	0.0438
4	4	4	0.05811	0.17632	0.0374
5	2	5	0.05695	0.17388	0.0340
6	3	4	0.06701	0.17103	0.0346
7	4	5	0.01335	0.04211	0.0
11	6	11	0.09798	0.19890	0.0
12	6	12	0.12291	0.25581	0.0
13	6	13	0.06615	0.13027	0.0
14	7	8	0.0	0.17615	0.0
15	7	9	0.0	0.11001	0.0
16	9	10	0.03181	0.08450	0.0
17	9	14	0.12711	0.27038	0.0
18	10	11	0.08205	0.19207	0.0
19	12	13	0.22092	0.19988	0.0
20	13	14	0.17093	0.34802	0.0

### DATA FOR 11 BUS SYSTEM

The 11 bus system is shown in Fig. A.4. The system data is taken from ref [4,7]. The relevant data are provided in following tables. Table A.14 gives bus data and Table A.15 gives Y matrix elements

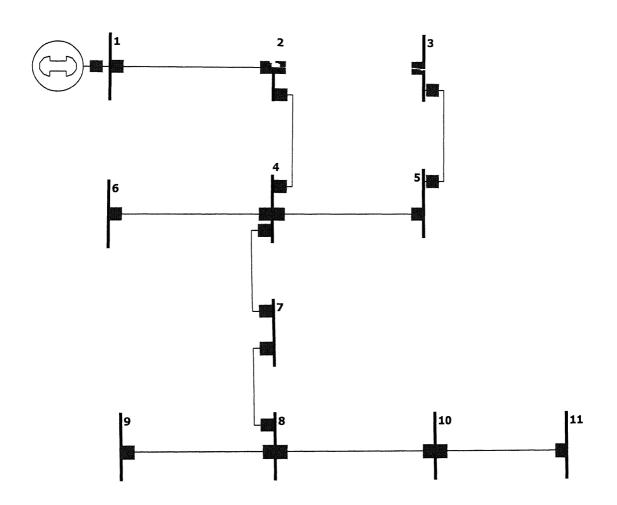


Fig A.4 11 bus ill-conditioned system

Table A.14 Bus data for 11 bus system

Bus Number	Voltage (p.u.)	Angle (radians)	Real power injection (p.u.)	Reactive power injection (p.u.)
1	1.024	0	0	0
2	1	0	0	0
3	1	0	-0.128	-0.062
4	1	0	0	0
5	1	0	-0.165	-0.080
6	1	0	-0.090	-0.068
7	1	0	0	0
8	1	0	0	0
9	1	0	-0.026	-0.009
10	1	0	0	0
11	1	0	-0.158	-0.057

Table A.15 Y bus matrix for 11 bus system

From Bus i	To bus j	Conductance G <sub>11</sub>	Susceptance B <sub>ij</sub>
		( <b>p.u.</b> )	(p.u.)
1	1	0.0	-14.939
1	2	O.O	14.148
2	2	12.051	-33.089
2	3	0.0	6.494
2	4	-12.051	13.197
3	3	2.581	-10.282
3	5	-2.5 81	3.789
4	4	12.642	-74.081
4	5	0. <b>O</b>	2.177
4	6	0. <b>O</b>	56.689
4	7	-O.5 92	0.786
5	5	2.581	-5.889
6	6	0. <b>Q</b>	-55.556
7	7	3.226	-4.304
7	8	-2.2 13	2.959
8	8	2.893	-5.468
8	9	138	1.379
8	10	-O.8 51	1.163
9	9	0.1 04	-1.042
10	10	1.346	-6.11
10	11	-0.374	3.742
11	11	O.2 <b>8</b> 3	-2.785

Base MVA=100

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